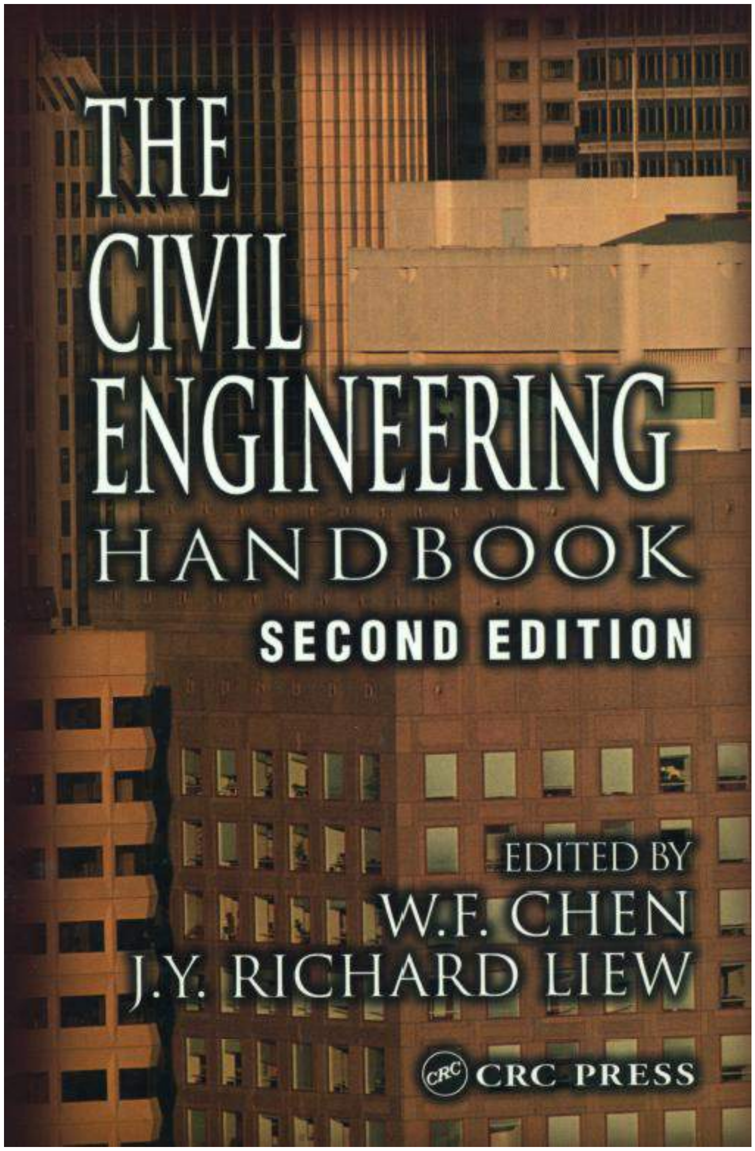

Reference

Fellenius, B. H., 2002. Foundations. Civil Engineering Handbook, Second Edition. Edited by W.F. Chen and J.Y.R. Liew, CRS Press, Section III. Geotechnical Engineering, Chapter 23 Foundations, pp. 23 1 through 23-36..

23
Foundations

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Before a foundation design can be undertaken, the associated soil profile must be well established. The soil profile is compiled from three cornerstones of information: borehole records with laboratory classification and testing, piezometer observations, and assessment of the overall geology at the site. Projects where construction difficulties, disputes, and litigation arise often have one thing in common: Copies of borehole field records were thought to determine the soil profile.

An essential part of the foundation design is to come up with a foundation type and size which will have acceptable values of deformation (settlement) and an adequate margin of safety to failure (the degree of engaging the soil strength). Deformation is a function of *change* of effective stress, and soil strength is *proportional* to effective stress. Therefore, all applications of foundation design start with determining the effective stress distribution of the soil around and below the foundation unit. The distribution then serves as basis for the design analysis.

23.1 Effective Stress

Effective stress is the total stress minus the **pore pressure** (the water pressure in the voids). The common method of calculating the effective stress, $\Delta\sigma'$, contributed by a soil layer is to multiply the buoyant unit weight, γ', of the soil with the layer thickness, Δh. Usually, the buoyant unit weight is determined as the bulk unit weight of the soil, γ_t, minus the unit weight of water, γ_w, which presupposes that there is no vertical gradient of water flow in the soil.

$$\Delta\sigma' = \gamma'\Delta h \tag{23.1a}$$

The effective stress at a depth, σ_z', is the sum of the contributions from the soil layers above.

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$$\sigma_z' = \Sigma(\gamma'\Delta h) \tag{23.1b}$$

However, most sites display vertical water gradients: either an upward flow, maybe even artesian (the head is greater than the depth from the ground surface), or a downward flow, and the buoyant unit weight is a function of the gradient, i, in the soil, as follows.

$$\gamma' = \gamma_t - \gamma_w(1 - i) \tag{23.1c}$$

The gradient is defined as the difference in head between two points divided by the distance the water has to flow between these two points. Upward flow gradient is negative and downward flow is positive. For example, if for a particular case of artesian condition the gradient is nearly equal to –1, then, the buoyant weight is nearly zero. Therefore, the effective stress is close to zero, too, and the soil has little or no strength. This is the case of so-called quicksand, which is not a particular type of sand, but a soil, usually a silty fine sand, subjected to a particular pore pressure condition.

The gradient in a nonhydrostatic condition is often awkward to determine. However, the difficulty can be avoided, because the effective stress is most easily determined by calculating the total stress and the pore water pressure separately. The effective stress is then obtained by simple subtraction of the latter from the former.

Note the difference in terminology — effective *stress* and pore *pressure* — which reflects the fundamental difference between forces in soil as opposed to in water. Soil stress is directional; that is, the stress changes depending on the orientation of the plane of action in the soil. In contrast, water pressure is omnidirectional, that is, independent of the orientation of the plane. The soil stress and water pressures are determined, as follows.

The *total vertical stress* (symbol σ_z) at a point in the soil profile (also called total overburden stress) is calculated as the stress exerted by a soil column with a certain weight, or bulk unit weight, and height (or the sum of separate values where the soil profile is made up of a series of separate soil layers having different bulk unit weights). The symbol for bulk unit weight is γ_t (the subscript t stands for "total," because the bulk unit weight is also called *total unit weight*).

$$\sigma_z = \gamma_t z \tag{23.2}$$

or

$$\sigma_z = \Sigma\Delta\sigma_z = \Sigma(\gamma_t \Delta h)$$

Similarly, the pore pressure (symbol u), if measured in a stand-pipe, is equal to the unit weight of water, γ_w, times the height of the water column, h, in the stand-pipe. (If the pore pressure is measured directly, the head of water is equal to the pressure divided by the unit weight of water, γ_w.)

$$u = \gamma_w h \tag{23.3}$$

The height of the column of water (the head) representing the water pressure is rarely the distance to the ground surface or, even, to the groundwater table. For this reason, the height is usually referred to as the *phreatic height* or the *piezometric height* to separate it from the depth below the groundwater table or depth below the ground surface.

The groundwater table is defined as the uppermost level of zero pore pressure. Notice that the soil can be saturated with water also above the groundwater table without pore pressure being greater than zero. Actually, because of capillary action, pore pressures in the partially saturated zone above the groundwater table may be negative.

The pore pressure distribution is determined by applying that (in stationary situations) the pore pressure distribution is linear in each individual soil layer, and, in pervious soil layers that are "sandwiched" between less pervious layers, the pore pressure is hydrostatic (that is, the vertical gradient is zero).

The *effective overburden stress* (symbol σ'_z) is then obtained as the difference between total stress and pore pressure.

$$\sigma'_z = \sigma_z - u_z = \gamma_t z - \gamma_w h \tag{23.4}$$

Usually, the geotechnical engineer provides the density (symbol ρ) instead of unit weight, γ. The unit weight is then the density times the gravitational constant, g. (For most foundation engineering purposes, the gravitational constant can be taken to be 10 m/s^2 rather than the overly exact value of 9.81 m/s^2.)

$$\gamma = \rho g \tag{23.5}$$

Many soil reports do not indicate the total soil density, ρ_t, only water content, w, and dry density, ρ'_d. For saturated soils, the total density can then be calculated as

$$\rho_t = \rho_d(1 + w) \tag{23.6}$$

The principles of effective stress calculation are illustrated by the calculations involved in the following soil profile: an upper 4 m thick layer of normally consolidated sandy silt, 17 m of soft, compressible, slightly overconsolidated clay, followed by 6 m of medium dense silty sand and a thick deposit of medium dense to very dense sandy ablation glacial till. The groundwater table lies at a depth of 1.0 m. For *original conditions*, the pore pressure is hydrostatically distributed throughout the soil profile. For *final conditions*, the pore pressure in the sand is not hydrostatically distributed, but artesian with a phreatic height above ground of 5 m, which means that the pore pressure in the clay is non-hydrostatic (but linear, assuming that the final condition is long term). The pore pressure in the glacial till is also hydrostatically distributed. A 1.5 m thick earth fill is to be placed over a square area with a 36 m side. The densities of the four soil layers and the earth fill are: 2000 kg/m^3, 1700 kg/m^3, 2100 kg/m^3, 2200 kg/m^3, and 2000 kg/m^3, respectively.

Calculate the distribution of total and effective stresses as well as pore pressure underneath the center of the earth fill before and after placing the earth fill. Distribute the earth fill by means of the 2:1 method; that is, distribute the load from the fill area evenly over an area that increases in width and length by an amount equal to the depth below the base of the fill area.

Table 23.1 presents the results of the stress calculation for the example conditions. The calculations have been made with the Unisettle program [Goudreault and Fellenius, 1993] and the results are presented in the format of a "hand calculation" to ease verifying the computer calculations. Notice that performing the calculations at every meter depth is normally not necessary. The table includes a comparison between the non-hydrostatic pore pressure values and the hydrostatic, as well as the effect of the earth fill, which can be seen from the difference in the values of total stress for original and final conditions.

The stress distribution below the center of the loaded area was calculated by means of the 2:1 method. However, the 2:1 method is rather approximate and limited in use. Compare, for example, the vertical stress below a loaded footing that is either a square or a circle with a side or diameter of B. For the same contact stress, q_0, the 2:1 method, strictly applied to the side and diameter values, indicates that the vertical distributions of stress, $[q_z = q_0/(B + z)^2]$, are equal for the square and the circular footings. Yet, the total applied load on the square footing is $4/\pi = 1.27$ times larger than the total load on the circular footing. Therefore, if applying the 2:1 method to circles and other non-rectangular areas, they should be modeled as a rectangle of an equal size ("equivalent") area. Thus, a circle is modeled as an equivalent square with a side equal to the circle radius times $\sqrt{\pi}$.

More important, the 2:1 method is inappropriate to use for determining the stress distribution along a vertical line below a point at any other location than the center of the loaded area. For this reason, it can not be used to combine stress from two or more loaded areas unless the areas have the same center. To calculate the stresses induced from more than one loaded area and/or below an off-center location, more elaborate methods, such as the Boussinesq distribution (Chapter 20) are required.

A footing is usually placed in an excavation and fill is often placed next to the footing. When calculating the stress increase from one or more footing loads, the changes in effective stress from the excavations

TABLE 23.1 Stress Distribution (2:1 Method) at Center of Earth Fill

	Original Condition (no earth fill)			Final Condition (with earth fill)		
Depth (m)	σ_t (kPa)	u (kPa)	σ' (kPa)	σ_t (kPa)	u (kPa)	σ' (kPa)
			Layer 1 Sandy Silt $\rho = 2000$ kg/m³			
0.00	0.0	0.0	0.0	30.0	0.0	30.0
1.00(GWT)	20.0	0.0	20.0	48.4	0.0	48.4
2.00	40.0	10.0	30.0	66.9	10.0	56.9
3.00	60.0	20.0	40.0	85.6	20.0	65.6
4.00	80.0	30.0	50.0	104.3	30.0	74.3
			Layer 2 Soft Clay $\rho = 1700$ kg/m³			
4.00	80.0	30.0	50.0	104.3	30.0	74.3
5.00	97.0	40.0	57.0	120.1	43.5	76.6
6.00	114.0	50.0	64.0	136.0	57.1	79.0
7.00	131.0	60.0	71.0	152.0	70.6	81.4
8.00	148.0	70.0	78.0	168.1	84.1	84.0
9.00	165.0	80.0	85.0	184.2	97.6	86.6
10.00	182.0	90.0	92.0	200.4	111.2	89.2
11.00	199.0	100.0	99.0	216.6	124.7	91.9
12.00	216.0	110.0	106.0	232.9	138.2	94.6
13.00	233.0	120.0	113.0	249.2	151.8	97.4
14.00	250.0	130.0	120.0	265.6	165.3	100.3
15.00	267.0	140.0	127.0	281.9	178.8	103.1
16.00	284.0	150.0	134.0	298.4	192.4	106.0
17.00	301.0	160.0	141.0	314.8	205.9	109.0
18.00	318.0	170.0	148.0	331.3	219.4	111.9
19.00	335.0	180.0	155.0	347.9	232.9	114.9
20.00	352.0	190.0	162.0	364.4	246.5	117.9
21.00	369.0	200.0	169.0	381.0	260.0	121.0
			Layer 3 Silty Sand $\rho = 2100$ kg/m³			
21.00	369.0	200.0	169.0	381.0	260.0	121.0
22.00	390.0	210.0	180.0	401.6	270.0	131.6
23.00	411.0	220.0	191.0	422.2	280.0	142.2
24.00	432.0	230.0	202.0	442.8	290.0	152.8
25.00	453.0	240.0	213.0	463.4	300.0	163.4
26.00	474.0	250.0	224.0	484.1	310.0	174.1
27.00	495.0	260.0	235.0	504.8	320.0	184.8
			Layer 4 Ablation Till $\rho = 2200$ kg/m³			
27.00	495.0	260.0	235.0	504.8	320.0	184.8
28.00	517.0	270.0	247.0	526.5	330.0	196.5
29.00	539.0	280.0	259.0	548.2	340.0	208.2
30.00	561.0	290.0	271.0	569.9	350.0	219.9
31.00	583.0	300.0	283.0	591.7	360.0	231.7
32.00	605.0	310.0	295.0	613.4	370.0	243.4
33.00	627.0	320.0	307.0	635.2	380.0	255.2

Note: Calculations by means of UNISETTLE.

and fills must be included which therefore precludes the use of the 2:1 method (unless all such excavations and fills surround and are concentric with the footing).

A small diameter footing, of about 1 meter width, can normally be assumed to distribute the stress evenly over the footing contact area. However, this cannot be assumed to be the case for wider footings. The Boussinesq distribution assumes ideally flexible footings (and ideally elastic soil). Kany [1959] showed that below a so-called characteristic point, the vertical stress distribution is equal for flexible and stiff footings. The characteristic point is located at a distance of 0.37B and 0.37L from the center of a rectangular footing of sides B and L and at a radius of 0.37R from the center of a circular footing of radius R. When applying Boussinesq method of stress distribution to regularly shaped footings, the stress below this point is normally used rather than the stress below the center of the footing.

Calculation of the stress distribution below a point within or outside the footprint of a footing by means of the Boussinesq method is time-consuming, in particular if the stress from several loaded areas are to be combined. The geotechnical profession has for many years simplified the calculation effort by using nomograms over "influence values for vertical stress" at certain locations within the footprint of a footing. The Newmark influence chart [Newmark, 1935, 1942] is a classic. The calculations are still rather time consuming. However, since the advent of the computer, several computer programs are available which greatly simplify and speed up the calculation effort — for example, Unisettle by Goudreault and Fellenius [1993].

23.2 Settlement of Foundations

A **foundation** is a constructed unit that transfers the load from a superstructure to the ground. With regard to vertical loads, most foundations receive a more or less concentrated load from the structure and transfer this load to the soil underneath the foundation, distributing the load as a stress over a certain area. Part of the soil structure interaction is then the condition that the stress must not give rise to a deformation of the soil in excess of what the superstructure can tolerate.

The amount of deformation for a given contact stress depends on the distribution of the stress over the affected soil mass in relation to the existing stress (the imposed change of effective stress) and the compressibility of the soil layer. The change of effective stress is the difference between the initial (original) effective stress and the final effective stress, as illustrated in Table 23.1. The stress distribution has been discussed in the foregoing. The compressibility of the soil mass can be expressed in either simple or complex terms. For simple cases, the soil can be assumed to have a linear stress–strain behavior and the compressibility can be expressed by an elastic modulus.

Linear stress–strain behavior follows Hooke's law:

$$\varepsilon = \frac{\Delta\sigma'}{E} \tag{23.7}$$

where ε = induced strain in a soil layer
$\Delta\sigma'$ = imposed change of effective stress in the soil layer
E = elastic modulus of the soil layer

Often the elastic modulus is called *Young's modulus*. Strictly speaking, however, Young's modulus is the modulus for when lateral expansion is allowed, which may be the case for soil loaded by a small footing, but not when loading a larger area. In the latter case, the lateral expansion is constrained. The constrained modulus, D, is larger than the E-modulus. The constrained modulus is also called the *oedometer modulus*. For ideally elastic soils, the ratio between D and E is:

$$\frac{D}{E} = \frac{(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \tag{23.8}$$

where ν = Poisson's ratio. For example, for a soil material with a Poisson's ratio of 0.3, a common value, the constrained modulus is 35% larger than the Young's modulus. (Notice also that the concrete inside a concrete-filled pipe pile behaves as a constrained material as opposed to the concrete in a concrete pile. Therefore, when analyzing the deformation under load, use the constrained modulus for the former and the Young's modulus for the latter.)

The deformation of a soil layer, s, is the strain, ε, times the thickness, h, of the layer. The settlement, S, of the foundation is the sum of the deformations of the soil layers below the foundation.

$$S = \sum s = \sum (\varepsilon h) \tag{23.9}$$

However, stress–strain behavior is nonlinear for most soils. The nonlinearity cannot be disregarded when analyzing compressible soils, such as silts and clays; that is, the elastic modulus approach is not appropriate for these soils. Nonlinear stress–strain behavior of compressible soils is conventionally modeled by Eq. (23.10):

$$\varepsilon = \frac{C_c}{1 + e_0} \lg \frac{\sigma_1'}{\sigma_0'} \tag{23.10}$$

where σ_0' = original (or initial) effective stress
σ_1' = final effective stress
C_c = compression index
e = void ratio

The compression index and the void ratio parameters C_c and e_0 are determined by means of oedometer tests in the laboratory.

If the soil is overconsolidated, that is, consolidated to a stress (called "preconsolidation stress") larger than the existing effective stress, Eq. (23.10) changes to

$$\varepsilon = \frac{1}{1 + e_0}\left[C_{cr} \lg \frac{\sigma_p'}{\sigma_0'} + C_c \lg \frac{\sigma_1'}{\sigma_p'}\right] \tag{23.11}$$

where σ_p' = preconsolidation stress and C_{cr} = recompression index.

Thus, in conventional engineering practice of settlement design, two compression parameters need to be established. This is an inconvenience that can be avoided by characterizing the soil with the ratios $C_c/(1 + e_0)$ and $C_{cr}/(1 + e_0)$ as single parameters (usually called compression ratio, CR, and recompression ratio, RR, respectively), but few do. Actually, on surprisingly many occasions, geotechnical engineers only report the C_c parameter — neglecting to include the e_0 value — or worse, report the C_c from the oedometer test and the e_0 from a different soil specimen than used for determining the compression index! This is not acceptable, of course. The undesirable challenge of ascertaining what C_c value goes with what e_0 value is removed by using the Janbu tangent modulus approach instead of the C_c and e_0 approach, applying a modulus number, m, instead.

The Janbu tangent modulus approach, proposed by Janbu [1963, 1965, 1967] and referenced by the Canadian Foundation Engineering Manual, (CFEM) [Canadian Geotechnical Society, 1985], applies the same basic principle of nonlinear stress–strain behavior to all soils, clays as well as sand. By this method, the relation between stress and strain is a function of two nondimensional parameters which are unique for a soil: a stress exponent, j, and a modulus number, m.

In cohesionless soils, $j > 0$, the following simple formula governs:

$$\varepsilon = \frac{1}{mj}\left[\left(\frac{\sigma_1'}{\sigma_r'}\right)^j - \left(\frac{\sigma_0'}{\sigma_r'}\right)^j\right] \tag{23.12}$$

where ε = strain induced by increase of effective stress
σ_0' = the original effective stress
σ_1' = the final effective stress
j = the stress exponent
m = the modulus number, which is determined from laboratory and/or field testing
σ_r' = a reference stress, a constant, which is equal to 100 kPa (= 1 tsf = 1 kg/cm^2)

In an essentially cohesionless, sandy or silty soil, the stress exponent is close to a value of 0.5. By inserting this value and considering that the reference stress is equal to 100 kPa, Eq. (23.12) is simplified to

$$\varepsilon = \frac{1}{5m}\left(\sqrt{\sigma_1'} - \sqrt{\sigma_0'}\right). \tag{23.13a}$$

Notice that Eq. (23.13a) is *not* independent of the choice of units; the stress values must be inserted in kPa. That is, a value of 5 MPa is to be inserted as the number 5000 and a value of 300 Pa as the number 0.3. In English units, Eq. (23.13a) becomes

$$\varepsilon = \frac{2}{m}\left(\sqrt{\sigma_1'} - \sqrt{\sigma_0'}\right). \tag{23.13b}$$

Again, the equation is not independent of units: Because the reference stress converts to 1.0 tsf, Eq. (23.13b) requires that the stress values be inserted in units of tsf.

If the soil is overconsolidated and the final stress exceeds the preconsolidation stress, Eqs. (23.13a) and (23.13b) change to

$$\varepsilon = \frac{1}{5m_r}\left(\sqrt{\sigma_p'} - \sqrt{\sigma_0'}\right) + \frac{1}{5m}\left(\sqrt{\sigma_1'} - \sqrt{\sigma_p'}\right) \tag{23.14a}$$

$$\varepsilon = \frac{2}{m_r}\left(\sqrt{\sigma_p'} - \sqrt{\sigma_0'}\right) + \frac{2}{m}\left(\sqrt{\sigma_1'} - \sqrt{\sigma_p'}\right) \tag{23.14b}$$

where σ_0' = original effective stress (kPa or tsf)
σ_p' = preconsolidation stress (kPa or tsf)
σ_1' = final effective stress (kPa or tsf)
m = modulus number (dimensionless)
m_r = recompression modulus number (dimensionless)

Equation (23.14a) requires stress units in kPa and Eq. (23.14b) stress units in tsf.

If the imposed stress does not result in a new (final) stress that exceeds the preconsolidation stress, Eqs. (23.13a) and (23.13b) become

$$\varepsilon = \frac{1}{5m_r}\left(\sqrt{\sigma_1'} - \sqrt{\sigma_0'}\right) \tag{23.15a}$$

$$\varepsilon = \frac{2}{m_r}\left(\sqrt{\sigma_1'} - \sqrt{\sigma_0'}\right) \tag{23.15b}$$

Equation (23.15a) requires stress units in kPa and Eq. (23.15b) units in tsf.

In cohesive soils, the stress exponent is zero, $j = 0$. Then, in a normally consolidated cohesive soil:

$$\varepsilon = \frac{1}{m}\ln\left(\frac{\sigma_1'}{\sigma_0'}\right) \tag{23.16}$$

and in an overconsolidated cohesive soil

$$\varepsilon = \frac{1}{m_r}\ln\left(\frac{\sigma_p'}{\sigma_0'}\right) + \frac{1}{m}\ln\left(\frac{\sigma_1'}{\sigma_p'}\right) \tag{23.17}$$

TABLE 23.2 Typical and Normally Conservative Modulus Numbers

Soil Type		Modulus Number	Stress Exp.
Till, very dense to dense		1000–300	(j = 1)
Gravel		400–40	(j = 0.5)
Sand	Dense	400–250	(j = 0.5)
	Compact	250–150	"
	Loose	150–100	"
Silt	Dense	200–80	(j = 0.5)
	Compact	80–60	"
	Loose	60–40	"
		Clays	
Silty clay	Hard, stiff	60–20	(j = 0)
and	Stiff, firm	20–10	"
clayey silt	Soft	10–5	"
Soft marine clays and organic clays		20–5	(j = 0)
Peat		5–1	(j = 0)

Notice that the ratio (σ'_p/σ'_0) is equal to the overconsolidation ratio, OCR. Of course, the extent of overconsolidation may also be expressed as a fixed stress-unit value, $\sigma'_p - \sigma'_0$.

If the imposed stress does not result in a new stress that exceeds the preconsolidation stress, Eq. (23.17) becomes

$$\varepsilon = \frac{1}{m_r} \ln\left(\frac{\sigma'_1}{\sigma'_0}\right) \tag{23.18}$$

By means of Eqs. (23.10) through (23.18), settlement calculations can be performed without resorting to simplifications such as that of a constant elastic modulus. Apart from having knowledge of the original effective stress, the increase of stress, and the type of soil involved, without which knowledge no reliable settlement analysis can ever be made, the only soil parameter required is the modulus number. The modulus numbers to use in a particular case can be determined from conventional laboratory testing, as well as *in situ* tests. As a reference, Table 23.2 shows a range of normally conservative values, in particular for the coarse-grained soil types, which are typical of various soil types (quoted from the CFEM [Canadian Geotechnical Society, 1985]).

In a cohesionless soil, where previous experience exists from settlement analysis using the elastic modulus approach — Eqs. (23.7) and (23.9) — a direct conversion can be made between E and m, which results in Eq. (23.19a).

$$m = \frac{E}{5(\sigma'_1 + \sigma'_0)} = \frac{E}{10\bar{\sigma}'} \tag{23.19a}$$

Equation (23.19a) is valid when the calculations are made using SI units. Notice, stress and E-modulus must be expressed in the same units, usually kPa. When using English units (stress and E-modulus in tsf), Eq. (23.19b) applies.

$$m = \frac{2E}{(\sigma'_1 + \sigma'_0)} = \frac{E}{\bar{\sigma}'} \tag{23.19b}$$

Notice that most natural soils have aged and become overconsolidated with an overconsolidation ratio, OCR, that often exceeds a value of 2. For clays and silts, the recompression modulus, m_r, is often five to ten times greater than the virgin modulus, m, listed in Table 23.2.

The conventional C_c and e_0 method and the Janbu modulus approach are identical in a cohesive soil. A direct conversion factor as given in Eq. (23.20) transfers values of one method to the other.

$$m = \ln 10\left(\frac{1+e_0}{C_c}\right) = 2.30\left(\frac{1+e_0}{C_c}\right) \tag{23.20}$$

Designing for settlement of a foundation is a prediction exercise. The quality of the prediction — that is, the agreement between the calculated and the actual settlement value — depends on how accurately the soil profile and stress distributions applied to the analysis represent the site conditions, and how closely the loads, fills, and excavations considered resemble those actually occurring. The quality depends also, of course, on the quality of the soil parameters used as input to the analysis. Soil parameters for cohesive soils depend on the quality of the sampling and laboratory testing. Clay samples tested in the laboratory should be from carefully obtained "undisturbed" piston samples. Paradoxically, the more disturbed the sample is, the less compressible the clay appears to be. The error which this could cause is to a degree "compensated for" by the simultaneous apparent reduction in the overconsolidation ratio. Furthermore, high-quality sampling and oedometer tests are costly, which limits the amounts of information procured for a routine project. The designer usually runs the tests on the "worst" samples and arrives at a conservative prediction. This is acceptable, but only too often is the word "conservative" nothing but a disguise for the more appropriate terms of "erroneous" and "unrepresentative" and the end results may not even be on the "safe side."

Non-cohesive soils cannot easily be sampled and tested. Therefore, settlement analysis of foundations in such soils must rely on empirical relations derived from *in situ* tests and experience values. Usually, these soils are less compressible than cohesive soils and have a more pronounced overconsolidation. However, considering the current tendency toward larger loads and contact stresses, foundation design must prudently address the settlement expected in these soils as well. Regardless of the methods that are used for prediction of the settlement, it is necessary to refer the results of the analyses back to basics. That is, the settlement values arrived at in the design analysis should be evaluated to indicate what corresponding compressibility parameters (Janbu modulus numbers) they represent for the actual soil profile and conditions of effective stress and load. This effort will provide a check on the reasonableness of the results as well as assist in building up a reference database for future analyses.

Time-Dependent Settlement

Because soil solids compress very little, settlement is mostly the result of a change of pore volume. Compression of the solids is called *initial compression*. It occurs quickly and it is usually considered elastic in behavior. In contrast, the change of pore volume will not occur before the water occupying the pores is squeezed out by an increase of stress. The process is rapid in coarse-grained soils and slow in fine-grained soils. In fine clays, the process can take a longer time than the life expectancy of the building, or of the designing engineer, at least. The process is called *consolidation* and it usually occurs with an increase of both undrained and drained soil shear strength. By analogy with heat dissipation in solid materials, the Terzaghi consolidation theory indicates simple relations for the time required for the consolidation. The most commonly applied theory builds on the assumption that water is able to drain out of the soil at one surface boundary and not at all at the opposite boundary. The consolidation is fast in the beginning, when the driving pore pressures are greater, and slows down with time. The analysis makes use of the relative amount of consolidation obtained at a certain time, called *average degree of consolidation*, which is defined as follows:

$$\bar{U} = \frac{S_t}{S_f} = 1 - \frac{\bar{u}_t}{\bar{u}_0} \tag{23.21}$$

where $\overline{U}$ = *average* degree of consolidation
S_t = settlement at time t
S_f = final settlement at full consolidation
$\bar{u}_t$ = *average* pore pressure at time t
$\bar{u}_0$ = initial *average* pore pressure (on application of the load; time $t = 0$)

Notice that the pore pressure varies throughout the soil layer and Eq. (23.21) assumes average values. In contrast, the settlement values are not the average, but the accumulated values.

The time for achieving certain degree consolidation is then, as follows:

$$t = T_v \frac{H^2}{c_v} \tag{23.22}$$

where t = time to obtain a certain degree of consolidation
T_v = a dimensionless time coefficient
c_v = coefficient of consolidation
H = length of the longest drainage path

The time coefficient, T_v, is a function of the type of pore pressure distribution. Of course, the shape of the distribution affects the average pore pressure values and a parabolic shape is usually assumed. The coefficient of consolidation is determined in the laboratory oedometer test (some *in situ* tests can also provide c_v values) and it can rarely be obtained more accurately than within a ratio range of 2 or 3. The length of the longest drainage path, H, for a soil layer that drains at both surface boundaries is half the layer thickness. If drainage only occurs at one boundary, H is equal to the full layer thickness. Naturally, in layered soils, the value of H is difficult to ascertain.

Approximate values of T_v for different degrees of consolidation are given below. For more exact values and values to use when the pore pressure distribution is different, see, for example, Holtz and Kovacs [1981].

U(%)	25	50	70	80	90	100
T_v	0.05	0.20	0.30	0.40	0.60	1.00

In partially saturated soils, consolidation determined from observed settlement is initially seemingly rapid, because gas (air) will readily compress when subjected to an increase of pressure. This settlement is often mistaken for the initial compression of the grain solids. However, because the pore pressure will not diminish to a similar degree, initial consolidation determined from observed pore pressures will not appear to be as large. In these soils and in seemingly saturated soils that have a high organic content, gas is present as bubbles in the pore water, and the bubbles will compress readily. Moreover, some of the gas may go into solution in the water as a consequence of the pressure increase. Inorganic soils below the groundwater surface are usually saturated and contain no gas. In contrast, organic soils will invariably contain gas in the form of small bubbles (as well as gas dissolved in the water, which becomes free gas on release of confining pressure when sampling the soil) and these soils will appear to have a fast initial consolidation. Toward the end of the consolidation process, when the pore pressure has diminished, the bubbles will return to the original size and the consolidation process will appear to have slowed.

Generally, the determination — prediction — of the time for a settlement to develop is filled with uncertainty and it is very difficult to reliably estimate the amount of settlement occurring within a specific time after the load application. The prediction is not any easier when one has to consider the development during the build-up of the load. For details on the subject, see Ladd [1991].

The rather long consolidation time in clay soils can be shortened considerably by means of vertical drains. Vertical drains installed at spacings ranging from about 1.2 m through 2.0 m have been very successful in accelerating consolidation from years to months. In the past, vertical drains consisted of sand drains and installation disturbance in some soils often made the drains cause more problems than

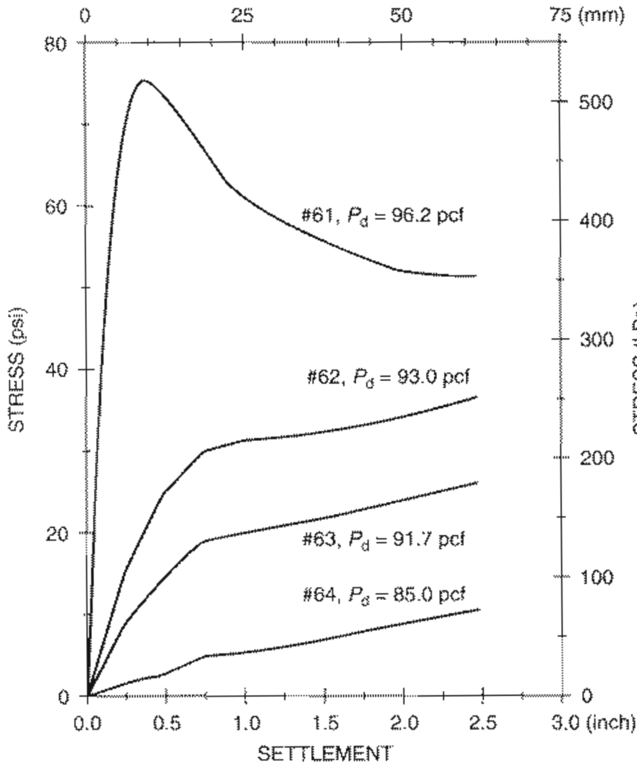

FIGURE 23.1 Contact stress vs. settlement of 150-mm footings. (*Source:* Vesic, 1967.)

in normally consolidated clay showed load–movement curves where the load increased to a distinct peak value — bearing capacity failure — indicating that the capacity (not the settlement) of a footing in clay is independent of the footing size.

The behavior of footing in clay differs from the behavior of footings in sand, however. Figure 23.1 presents results from loading tests on a 150-mm diameter footing in dry sand of densities varying from very dense to loose. In the dense sand, a peak value is evident. In less dense sands, no such peak is found.

The capacity and the load movement of a footing in sand are almost directly proportional to the footing size. This is illustrated in Fig. 23.2, which shows some recent test results on footings of different size in a fine sand. Generally, eccentric loading, inclined loading, footing shape, and foundation depth influence the behavior of footings. Early on, Terzaghi developed the theoretical explanations to observed behaviors into a "full bearing capacity formula," as given in Eq. (23.24a) and applicable to a continuous footing:

$$r_u = c'N_c + q'(N_q - 1) + 0.5B\,\gamma' N_\gamma \tag{23.24a}$$

where r_u = ultimate unit resistance of the footing
c' = effective cohesion intercept
B = footing width
q' = overburden effective stress at the foundation level
γ' = average effective unit weight of the soil below the foundation
N_c, N_q, N_γ = nondimensional bearing capacity factors

The **bearing capacity** factors are a function of the effective friction angle of the soil. Such factors were first originated by Terzaghi, later modified by Meyerhof, Berezantsev, and others. As presented in the *Canadian Foundation Engineering Manual* [Canadian Geotechnical Society, 1985], the bearing capacity factors are somewhat interrelated, as follows.

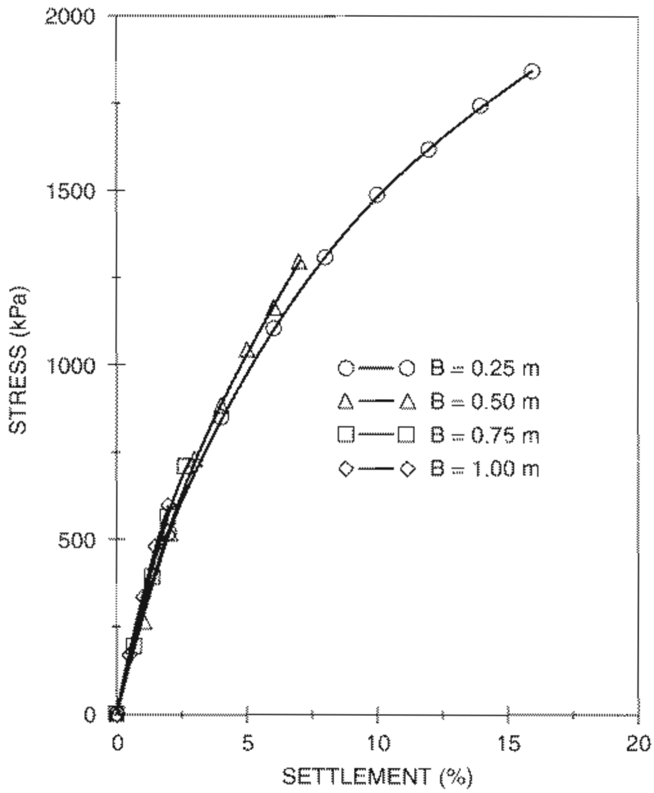

FIGURE 23.2 Stress vs. normalized settlement. (Data from Ismael, 1985.)

$$N_q = (e^{\pi \tan\varphi'})\left(\frac{1 + \sin\varphi'}{1 - \sin\varphi'}\right) \qquad \varphi' \rightarrow 0 \quad N_q \rightarrow 1 \tag{23.24b}$$

$$N_c = (N_q - 1)(\cot\varphi') \qquad \varphi' \rightarrow 0 \quad N_c \rightarrow 5.14 \tag{23.24c}$$

$$N_\gamma = 1.5(N_q - 1)(\tan\varphi') \qquad \varphi' \rightarrow 0 \quad N_\gamma \rightarrow 0 \tag{23.24d}$$

For friction angles larger than about 37°, the bearing capacity factors increase rapidly and the formula loses in relevance.

For a footing of width B subjected to a load Q, the applied contact stress is q (= Q/B) per unit length and the applied contact stress mobilizes an equally large soil resistance, r. Of course, the soil resistance can not exceed the strength of the soil. Equation (23.24a) indicates the maximum available (ultimate) resistance, r_u. In the design of a footing for bearing capacity, the applied load is only allowed to reach a certain portion of the ultimate resistance. That is, as is the case for all foundation designs, the design must include a margin of safety against failure. In most geotechnical applications, this margin is achieved by applying a factor of safety defined as the available soil strength divided by the mobilized shear. The available strength is either cohesion, c, friction, tan φ, or both combined. (Notice that friction is not the friction angle, φ, but its tangent, tan φ). However, in bearing capacity problems, the factor of safety is usually defined somewhat differently and as given by Eq. (23.24e):

$$F_s = r_u / q_{\text{allow}} \tag{23.24e}$$

where F_s = factor of safety
r_u = ultimate unit resistance (unit bearing capacity)
q_{allow} = the allowable bearing stress

The factor of safety applied to the bearing capacity formula is usually recommended to be no smaller than 3.0, usually equal to 4.0. There is some confusion whether, in the bearing capacity calculated according to Eq. (23.24a), the relation ($N_q - 1$) should be used in lieu of N_q and, then, whether or not the allowable bearing stress should be the "net" stress, that is, the value exceeding the existing stress at the footing base. More importantly, however, is that the definition of factor of safety given by Eq. (23.24e) is not the same as the factor of safety applied to the shear strength, because the ultimate resistance determined by the bearing capacity formula includes several aspects other than soil shear strength, particularly so for foundations in soil having a substantial friction component. Depending on the details of each case, a value of 3 to 4 for the factor defined by Eq. (23.24e) corresponds, very approximately, to a factor of safety on shear strength in the range of 1.5 through 2.0.

In fact, the bearing capacity formula is wrought with much uncertainty and the factor of safety, be it 3 or 4, applied to a bearing capacity formula is really a "factor of ignorance" and does not always guarantee an adequate safety against failure. Therefore, in the design of footings, be it in clays or sands, the settlement analysis should be given more weight than the bearing capacity formula calculation.

Footings are rarely loaded only vertically and concentrically. Figure 23.3(b) illustrates the general case of a footing subjected to both inclined and eccentric load. Eq. (23.24a) changes to

$$r_u = s_c i_c c' N_c + s_q i_q q' N_q + s_\gamma i_\gamma 0.5 B' \gamma' N_\gamma \tag{23.24f}$$

where factors not defined earlier are

s_c, s_q, s_γ = nondimensional shape factors
i_c, i_q, i_γ = nondimensional inclination factors
B' = equivalent or effective footing width

The shape factors are

$$s_c = s_q = 1 + (B'/L')(N_q/N_c) \tag{23.24g}$$

where L' = equivalent or effective footing length.

$$s_\gamma = 1 - 0.4(B'/L') \tag{23.24h}$$

The inclination factors are

$$i_c = i_q = (1 - \delta/90°)^2 \tag{23.24i}$$

$$i_\gamma = (1 - \delta/\varphi')^2 \tag{23.24j}$$

An inclined load can have a significant reducing effect on the bearing capacity of a footing. Directly, first, as reflected by the inclination factor and then also because the resultant to the load on most occasions acts off center. An off-center load will cause increased stress, edge stress, on one side and a decreased stress on the opposing side. A large edge stress can be the starting point of a failure. In fact, most footings, when they fail, fail by tilting, which is an indication of excessive edge stress. To reduce the risk for failure, the bearing capacity formula (which assumes a uniform load) applies the term B' in Eq. (23.24f), the effective footing width, which is the width of a smaller footing having the resultant load in its center. That is, the calculated ultimate resistance is decreased because of the reduced width (γ component) and the applied stress is increased because it is calculated over the effective area [as $q = Q/(B'/L')$]. The approach is approximate and its application is limited to the requirement that the contact stress must not be reduced beyond a zero value at the opposite edge ("No tension at the heel"). This means that the resultant must fall within the middle third of the footing, or the eccentricity must not be greater than $B/6$.

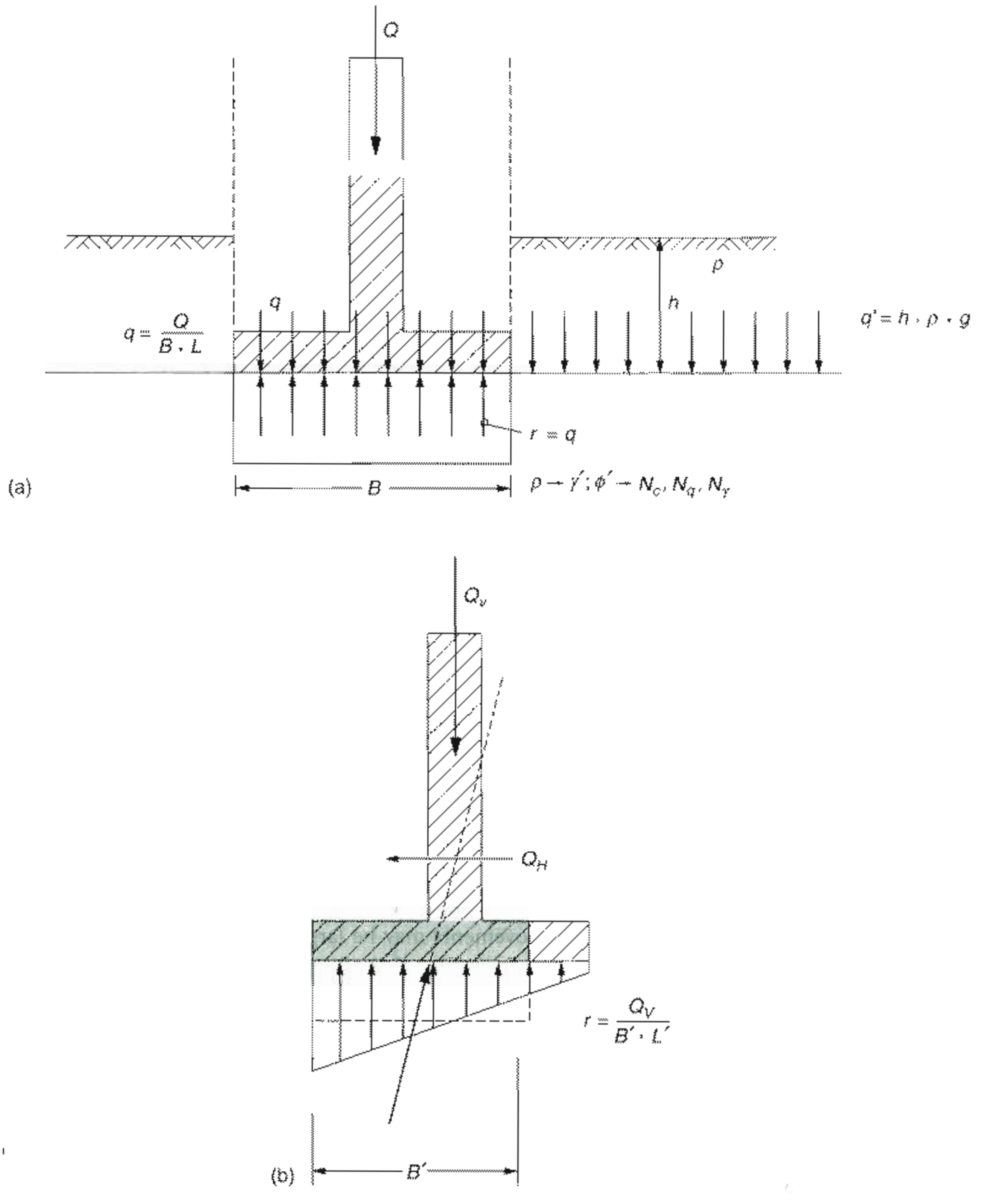

FIGURE 23.3 Input to the bearing capacity formula applied to a strip footing. (a) Concentric and vertical loading. (b) Vertical and horizontal loading.

When the load forms an angle with both sides of a footing or is eccentric in the directions of both the short and long sides of the footing, the calculation must be made twice, exchanging B' and L'.

The inclined load has a horizontal component and the calculation of a footing stability must check that the safety against sliding is sufficient. The calculation is simple and consists of determining the ratio between the horizontal and vertical loads, Q_h/Q_v. This ratio must be smaller than the soil strength (friction, $\tan\varphi'$, and/or cohesion, c') at the interface between the footing underside and the soil. Usually, a factor of safety of 1.5 through 1.8 applied to the soil strength is considered satisfactory.

In summary, the bearing capacity calculation of a footing is governed by the bearing capacity of a uniformly loaded equivalent footing, with a check for excessive edge stress (eccentricity) and safety against sliding. In some texts, an analysis of "overturning" is mentioned, which consists of taking the moment of forces at the edge of the footing and applying a factor of safety to the equilibrium. This is an incorrect approach, because long before the moment equilibrium has been reached, the footing fails due to excessive edge stress. (It is also redundant, because the requirement for the resultant to be located within the middle third takes care of the "overturning.") In fact, "overturning" failure will occur already at a calculated "factor of safety" as large as about 1.3 on the moment equilibrium. Notice that the factor of safety approach absolutely requires that the calculation of the stability of the structure indicates that it is stable also at a factor of safety very close to unity — theoretically stable, that is.

The bearing capacity calculations are illustrated in the example presented in Fig. 23.4. The example involves a 10.0 m long and 8.0 m high, vertically and horizontally loaded retaining wall (bridge abutment). The wall is placed on the surface of a "natural" coarse-grained soil and backfilled with a coarse material. A 1.0 m thick backfill is placed in front of the wall and over the front slab. The groundwater table lies close to the ground surface at the base of the wall. Figure 23.4(a) presents the data to include in an analysis.

In any analysis of a foundation case, a free-body diagram is necessary to ensure that all forces are accounted for in the analysis, such as shown in Fig. 23.4(b). Although the length of the wall is finite, it is normally advantageous to calculate the forces per unit length of the wall. To simplify the computations, the weight of the slab and the wall is ignored (or the slab weight is assumed included in the soil weights, and the weight of the wall [stem] is assumed included in the vertical load applied to the top of the wall).

The vertical forces denoted Q_1 and Q_2 are the load on the back slab of the wall. The two horizontal forces denoted P_1 and P_2 are the active earth pressure forces acting on a fictitious wall rising from the heel of the back slab, which wall is the boundary of the free body. Because this fictitious wall is soil, there is no wall friction to consider in the earth pressure calculation. Naturally, earth pressure also acts on the footing stem (the wall itself). Here, however, wall friction does exist, rotating the earth pressure resultant from the horizontal direction. Because of compaction of the backfill and the inherent stiffness of the stem, the earth pressure coefficient to use for earth pressure against the stem is larger than active pressure coefficient. This earth pressure is of importance for the structural design of the stem and it is quite different from the earth pressure to consider in the stability analysis of the wall.

Figure 23.4(b) does not indicate any earth pressure in front of the wall. It would have been developed on the passive side (the design assumes that movements may be large enough to develop active earth pressure behind the wall, but not large enough to develop fully the passive earth pressure against the front of the wall). In many projects a more or less narrow trench for burying pipes and other conduits is often dug in front of the wall. This, of course, eliminates the passive earth pressure, albeit temporarily.

The design calculations show that the factors of safety against bearing failure and against sliding are 3.29 and 2.09, respectively. The resultant acts at a point on the base of the footing at a distance of 0.50 m from the center, which is smaller than the limit of 1.00 m. Thus, it appears as if the footing is safe and stable and the edge stress acceptable. However, a calculation result must always be reviewed in a "what if" situation. That is, what if for some reason the backfill in front of the wall were to be removed over a larger area? Well, this seemingly minor change results in a reduction of the calculated factor of safety to 0.69. The possibility that this fill is removed at some time during the life of the structure is real. Therefore — although under the given conditions for the design problem, the factor of safety for the footing is adequate — the structure may not be safe.

Some words of caution: As mentioned above, footing design must emphasize settlement analysis. The bearing capacity formula approach is very approximate and should never be taken as anything beyond a simple estimate for purpose of comparing a footing design to previous designs. When concerns for capacity are at hand, the capacity analysis should include calculation using results from *in situ* testing (piezocone penetrometer and pressuremeter). Finite element analysis may serve as a very useful tool provided that a proven soil model is applied. Critical design calculations should never be permitted to rely solely on information from simple borehole data and N values (SPT-test data) applied to bearing capacity formulas.

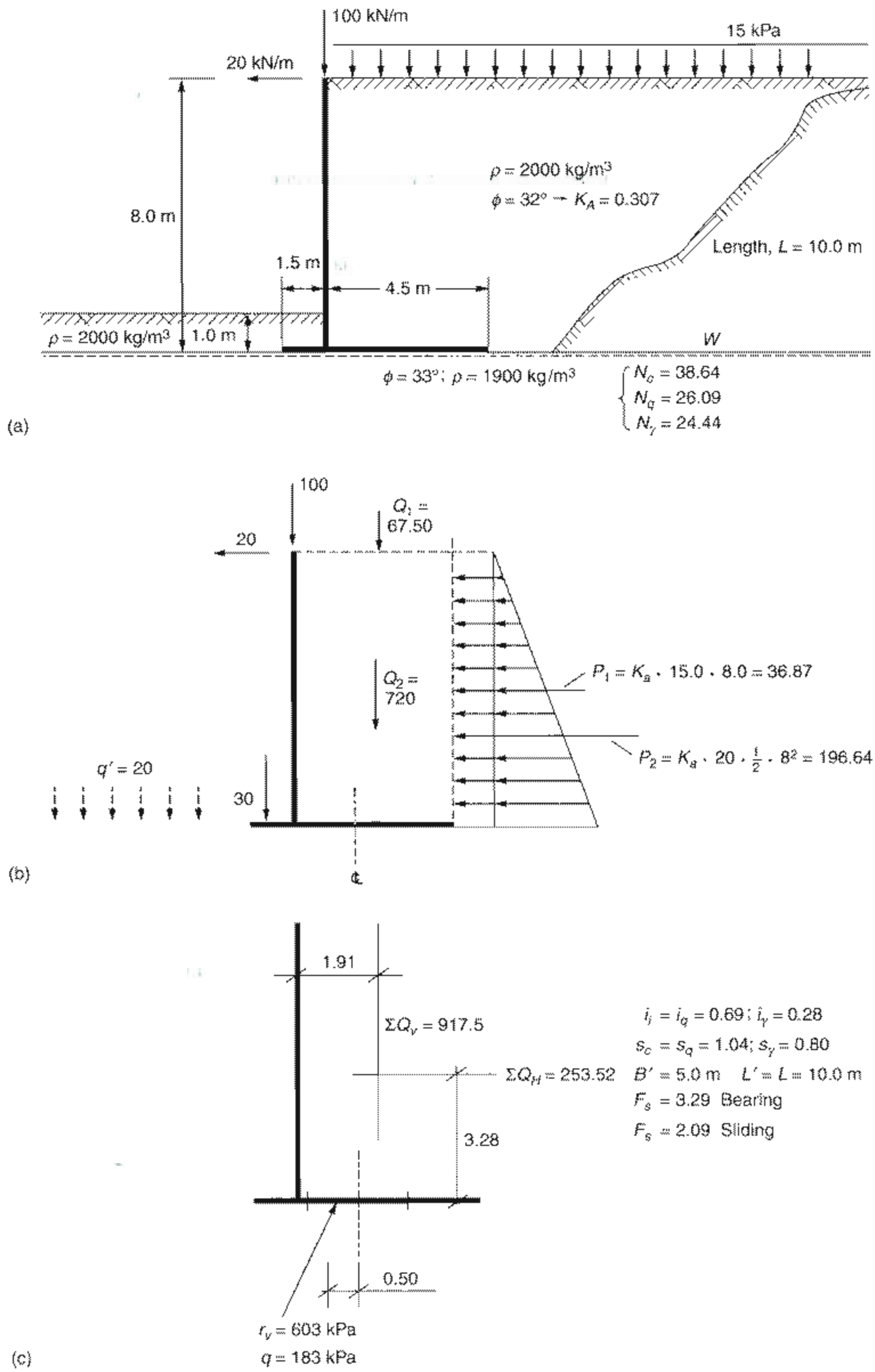

FIGURE 23.4 Bearing capacity example. (a) Problem background. (b) Free-body diagram. (c) Solution data.

23.4 Pile Foundations

Where using shallow foundations would mean unacceptable settlement, or where scour and other environmental risks exist which could impair the structure in the future, deep foundations are used. Deep foundations usually consist of piles, which are slender structural units installed by driving or by *in situ* construction methods through soft compressible soil layers into competent soils. Piles can be made of wood, concrete, or steel, or be composite, such as concrete-filled steel pipes or an upper concrete section connected to a lower steel or wood section. They can be round, square, hexagonal, octagonal, even triangular in shape, and straight shafted, step tapered, or conical. In order to arrive at a reliable design, the particulars of the pile must be considered, most important, the pile material and the method of construction.

Pile foundation design starts with an analysis of how the load applied to the pile head is transferred to the soil. This analysis is the basis for a settlement analysis, because in contrast to the design of shallow foundations, settlement analysis of piles cannot be separated from a load-transfer analysis. The load-transfer analysis is often called *static analysis* or *capacity analysis*. Total stress analysis using undrained shear strength (so-called α-method) has very limited application, because the load transfer between a pile and the soil is governed by effective stress behavior. In an effective stress analysis (also called β-method), the resistance is proportional to the effective overburden stress. Sometimes, an adhesion (cohesion) component is added. (The adhesion component is normally not applicable to driven piles, but may be useful for cast *in situ* piles). The total stress and effective stress approaches refer to both **shaft** and **toe resistances**, although the equivalent terms, "α-method" and "β-method" usually refer to shaft resistance, specifically.

Shaft Resistance

The general numerical relation for the unit shaft resistance, r_s, is

$$r_s = c' + \beta\sigma_z' \tag{23.25a}$$

The adhesion component, c', is normally set to zero for driven piles and Eq. (23.25a) then expresses that unit shaft resistance is directly proportional to the effective overburden stress.

The accumulated (total) shaft resistance, R_s, is

$$R_s = \int A_s r_s dz = \int A_s(c' + \beta\sigma_z')dz \tag{23.25b}$$

The beta coefficient varies with soil gradation, mineralogical composition, density, and soil strength within a fairly narrow range. Table 23.3 shows the approximate range of values to expect from basic soil types.

TABLE 23.3 Approximate Range of Beta Coefficients

Soil Type	Phi	Beta
Clay	25–30	0.25–0.35
Silt	28–34	0.27–0.50
Sand	32–40	0.30–0.60
Gravel	35–45	0.35–0.80

Toe Resistance

Also the unit toe resistance, r_t, is proportional to the effective stress, that is, the effective stress at the **pile toe** ($z = D$). The proportionality coefficient has the symbol N_t. Its value is sometimes stated to be of some relation to the conventional bearing capacity coefficient, N_q, but such relation is far from strict. The toe resistance, r_t, is

$$r_t = N_t\,\sigma_{z=D}' \tag{23.26a}$$

The total toe resistance, R_t, acting on a pile with a toe area equal to A_t is

$$R_t = A_t r_t = A_t N_t\,\sigma_{z=D}' \tag{23.26b}$$

In contrast to the β-coefficient, the toe coefficient, N_t, varies widely. Table 23.4 shows an approximate range of values for the four basic soil types.

TABLE 23.4 Approximate Range of N_t Coefficients

Soil Type	Phi	N_t
Clay	25–30	3–30
Silt	28–34	20–40
Sand	32–40	30–150
Gravel	35–45	60–300

Ultimate Resistance — Capacity

The capacity of the pile, Q_{ult} (alternatively, R_{ult}), is the sum of the shaft and toe resistances.

$$Q_{ult} = R_s + R_t \tag{23.27}$$

When the shaft and toe resistances are fully mobilized, the load in pile, Q_z, (as in the case of a static loading test brought to "failure") varies, as follows:

$$Q_z = Q_u - \int A_s \beta \sigma_z' dz = Q_u - R_s \tag{23.28}$$

Equation (23.28) is also called the *resistance distribution curve*. At the depth $z = D$, Eq. (23.28), of course, states that $Q_z = R_t$.

Notice that the commonly used term "ultimate capacity" is a misnomer and a tautology: a mix of the words "ultimate resistance" and "capacity". Although one cannot be mistaken about the meaning of ultimate capacity, the adjective should not be used, because it makes other adjectives seem proper, such as "load capacity," "allowable capacity," "design capacity," which are at best awkward and at worst misleading, because what is meant is not clear. Sometimes not even the person using these adjectives with "capacity" knows the meaning.

During service conditions, loads from the structure will be applied to the pile head via a pile cap. The loads are normally permanent (or "dead") loads, Q_d, and transient (or "live") loads Q_l. Not generally recognized is that even if soil settlement is small — too small to be noticeable — the soil will in the majority of cases move down in relation to the pile and in the process transfer load to the pile by negative skin friction. (The exception refers to piles in swelling soils and it is then limited to the length of pile in the swelling zone.) Already the extremely small relative movements always occurring between a pile shaft and the soil are sufficient to develop either shaft resistance or negative skin friction. Therefore, every pile develops an equilibrium of forces between, on the one side, the sum of dead load applied to the pile head, Q_d, and dragload, Q_n, induced by negative skin friction in the upper part of the pile, and, on the other side, the sum of positive shaft resistance and toe resistance in the lower part of the pile. The point of equilibrium, called the neutral plane, is the depth where the shear stress along the pile changes over from negative skin friction into positive shaft resistance. This is also where there is no relative displacement between the pile and the soil.

The key aspect of the foregoing is that the development of a neutral plane and negative skin friction is an always occurring phenomenon in piles and not only of importance in the context of large settlement of the soil around the piles.

Normally, the neutral plane lies below the midpoint of a pile. The extreme case is for a pile on rock, where the location of the neutral plane is at the bedrock elevation. For a dominantly shaft-bearing pile "floating" in a homogeneous soil with linearly increasing shear resistance, the neutral plane lies at a depth which is about equal to the lower third point of the pile embedment length.

The larger the toe resistance, the deeper the elevation of the neutral plane. And, the larger the dead load, the shallower the elevation of the neutral plane.

The load distribution in the pile during long-term conditions down to the neutral plane is given by the following load-transfer relation. [Below the neutral plane, Q_z follows Eq. (23.28).]

$$Q_z = Q_d + \int A_s q_n dz = Q_d + Q_n \tag{23.29}$$

The transition between the load-resistance curve [Eq. (23.27)] and the load-transfer curve [Eq. (23.28)] is in reality not the sudden kink the equations suggest, but a smooth transition over some length of pile, about 4 to 8 pile diameters above and below the neutral plane. (The length of this transition zone varies with the type of soil at the neutral plane.) Thus, the theoretically calculated value of the maximum load in the pile is higher than the real value. That is, it is easy to overestimate the magnitude of the dragload.

Critical Depth

Many texts suggest the existence of a so-called "critical depth" below which the shaft and toe resistances would be constant and independent of the increasing effective stress. This concept is a fallacy based on past incorrect interpretation of test data and should not be applied.

Effect of Installation

Whether a pile is installed by driving or by other means, the installation affects, disturbs, the soil. It is difficult to determine the magnitude of the shaft and toe resistances existing before the disturbance from the pile installation has subsided. For instance, presence of dissipating excess pore pressures causes uncertainity in the magnitude of the effective stress in the soil, ongoing strength gain due to reconsolidation is hard to estimate, etc. Such installation effects can take a long time to disappear, especially in clays. They can be estimated in an effective stress analysis using suitable assumptions as to the distribution of pore pressure along the pile at any particular time. Usually, to calculate the installation effect, a good estimate can be obtained by imposing excess pore pressures in the fine-grained soil layers, taking care that the pore pressure must not exceed the total overburden stress. By restoring the pore pressure values to the original conditions, which will prevail when the induced excess pore pressures have dissipated, the long-term capacity is found.

Residual Load

The dissipation of induced excess pore pressures is called *reconsolidation.* Reconsolidation after installation of a pile imposes load (residual load) in the pile by negative skin friction in the upper part of the pile, which is resisted by positive shaft resistance in the lower part of the pile and some toe resistance. The quantitative effect of including, as opposed to not to, the residual load in the analysis is that the shaft resistance becomes smaller and the toe resistance becomes larger. If the residual load is not recognized in the evaluation of results from a static loading test, totally erroneous conclusions will be drawn from the test: The shaft resistance appears larger than the true value, while the toe resistance appears correspondingly smaller, and if the resistance distribution is determined from a force gauge in the pile, zeroed at the start of the test, a "critical depth" will seem to exist. For more details on this effect and how to analyze the force gauge data to account for the residual load, see Altaee et al. [1992, 1993].

Analysis of Capacity for Tapered Piles

Many piles are not cylindrical or otherwise uniform in shape throughout the length. The most common example is the wood pile, which is conical in shape. Step-tapered piles are also common, consisting of two or more concrete-filled steel pipes of different diameters connected to each other, the larger above the smaller. Sometimes a pile can consist of a steel pipe with a conical section immediately above the pile toe, for example, the Monotube pile, which typically has a 25 feet (7.6 m) long conical end section, tapering the diameter down from 14 inches (355 mm) to 8 inches (203 mm). Piles can have an upper solid concrete section and a bottom H-pile extension.

For the step-tapered piles, obviously each "step" provides an extra resistance point, which needs to be considered in an analysis. (The GRLWEAP wave equation program [GRL, 1993], for example, can model a pile with one diameter change as having a second pile toe at the location of the step). Similarly, in a static analysis, each such step can be considered as a donut-shaped extra pile toe and assigned a corresponding toe resistance per Eq. (23.26). Each such extra toe resistance value is then added to the shaft resistance calculated using the actual pile diameter.

Piles with a continuous taper (conical piles) are less easy to analyze. Nordlund [1963] suggested a taper correction factor to use to increase the unit shaft resistance in sand for conical piles. The correction factor is a function of the taper angle and the soil friction angle. A taper angle of 1° (0.25 inch/foot) in a sand with a 35° friction angle would give a correction factor of about 4. At an angle of 0.5°, the factor would be about 2.

A more direct calculation method is to "step" the calculation in sub-layers of some thickness and at the bottom of each such sub-layer project the donut-shaped diameter change, which then is treated as an extra toe similar to the analysis of the step-taper pile. The shaft resistance is calculated using the mean diameter of the pile over the same "stepped" length. The shaft resistance over each such particular length consists of the sum of the shaft resistance and the extra-toe resistance. This method requires that a toe coefficient, N_t, be assigned to each soil layer.

The taper does not come into play for negative skin friction. This means that, when determining the dragload, the effect of the taper should be excluded. Below the neutral plane, however, the effect should be included. Therefore, the taper will influence the location of the neutral plane (because the taper increases the positive shaft resistance below the neutral plane).

Factor of Safety

In a pile design, one must distinguish between the design for bearing capacity and design for structural strength. The former is considered by applying a factor of safety to the pile capacity according to Eq. (23.27). The capacity is determined considering positive shaft resistance developed along the full length of the pile plus full toe resistance. Notice that no allowance is given for the dragload. If design is based on only theoretical analysis, the usual factor of safety is about 3.0. If based on the results of a loading test, static or dynamic, the factor of safety is reduced, depending on reliance on and confidence in the capacity value, and importance and sensitivity of the structure to foundation deformations. Factors of safety as low as 1.8 have then been used in actual design, but usually the lower bound is 2.0.

Design for structural strength applies to a factor of safety applied to the loads acting at the pile head and at the neutral plane. At the pile head, the loads consist of dead and live load combined with bending at the pile head, but no dragload. At the neutral plane, the loads consist of dead load and dragload, but no live load. Live load and dragload cannot occur at the same time and must, therefore, not be combined in the analysis. It is recommended that for *straight and undamaged piles* the allowable maximum load at the neutral plane be limited to 70% of the pile strength. For composite piles, such as concrete-filled pipe piles, the load should be limited to a value that induces a maximum compression strain of 1.0 millistrain into the pile with no material becoming stressed beyond 70% of its strength. The calculations are interactive inasmuch a change of the load applied to a pile will change the location of the neutral plane and the magnitude of the maximum load in the pile.

The third aspect in the design, calculation of settlement, pertains more to pile groups than to single piles. In extending the approach to a pile group, it must be recognized that a pile group is made up of a number of individual piles which have different embedment lengths and which have mobilized the toe resistance to a different degree. The piles in the group have two things in common, however: They are connected to the same stiff pile cap and, therefore, all pile heads move equally, and the piles must all have developed a neutral plane at the same depth somewhere down in the soil (long-term condition, of course).

Therefore, it is impossible to ensure that the neutral plane is common for the piles in the group, with the mentioned variation of length, etc., unless the dead load applied to the pile head from the cap differs between the piles. This approach can be used to discuss the variation of load within a group of stiffly connected piles. A pile with a longer embedment below the neutral plane or one having mobilized a larger toe resistance as opposed to other piles will carry a greater portion of the dead load on the group. On the other hand, a shorter pile, or one with a smaller toe resistance as opposed to other piles in the group, will carry a smaller portion of the dead load. If a pile is damaged at the toe, it is possible that the pile exerts a negative — pulling — force at the cap and thus increases the total load on the pile cap.

An obvious result of the development of the neutral plane is that no portion of the dead load is transferred to the soil via the pile cap, unless, of course, the neutral plane lies right at the pile cap and the entire pile group is failing.

Above the neutral plane, the soil moves down relative to the pile; below the neutral plane, the pile moves down into the soil. Therefore, at the neutral plane, the relative movement between the pile and the soil is zero, or, in other words, whatever the settlement of the soil that occurs at the neutral plane is equal to the settlement of the pile (the pile group) at the neutral plane. Between the pile head and the neutral plane, only deformation of the pile due to load occurs and this is usually minor. Therefore, settlement of the pile and the pile group is governed by the settlement of the soil at and below the neutral plane. The latter is influenced by the stress increase from the permanent load on the pile group and other causes of load, such as the fill. A simple method of calculation is to exchange the pile group for an equivalent footing of area equal to the area of the pile cap placed at the depth of the neutral plane. The load on the pile group load is then distributed as a stress on this footing calculating the settlement of this footing stress in combination with all other stress changes at the site, such as the earth fill, potential groundwater table changes, adjacent excavations, etc. Notice that the portion of the soil between the neutral plane and the pile toe depth is "reinforced" with the piles and, therefore, not very compressible. In most cases, the equivalent footing is best placed at the pile toe depth (or at the level of the average of the pile toe depths).

Empirical Methods for Determining Axial Pile Capacity

For many years, the *N*-index of standard penetration test has been used to calculate capacity of piles. Meyerhof [1976] compiled and rationalized some of the wealth of experience available and recommended that the capacity be a function of the *N*-index, as follows:

$$R = R_t + R_s = mNA_t + nNA_sD \tag{23.30}$$

where m = a toe coefficient
n = a shaft coefficient
N = *N*-index at the pile toe
N = average *N*-index along the pile shaft
A_t = pile toe area
A_s = unit shaft area; circumferential area
D = embedment depth

For values inserted into Eq. (23.30) using base SI units — that is, R in newton, D in meter, and A in m^2 — the toe and shaft coefficients, m and n, become:

$m = 400 \cdot 10^3$ for driven piles and $120 \cdot 10^3$ for bored piles (N/m^2)
$n = 2 \cdot 10^3$ for driven piles and $1 \cdot 10^3$ for bored piles (N/m^3)

For values inserted into Eq. (23.30) using English units with R in ton, D in feet, and A in ft², the toe and shaft coefficients, m and n, become:

$m = 4$ for driven piles and 1.2 for bored piles (N/m^2)
$n = 0.02$ for driven piles and 0.01 for bored piles (N/m^3)

The standard penetration test (SPT) is a subjective and highly variable test. The test and the *N*-index have substantial qualitative value, but should be used only very cautiously for quantitative analysis. The *Canadian Foundation Engineering Manual* [Canadian Geotechnical Society, 1985] includes a listing of the numerous irrational factors influencing the *N*-index. However, when the use of the *N*-index is considered with the sample of the soil obtained and related to a site- and area-specific experience, the crude and decried SPT test does not come out worse than other methods of analyses.

The static cone penetrometer resembles a pile. There is shaft resistance in the form of so-called local friction measured immediately above the cone point, and there is toe resistance in the form of the directly applied and measured cone-point pressure.

When applying cone penetrometer data to a pile analysis, both the local friction and the point pressure may be used as direct measures of shaft and toe resistances, respectively. However, both values can show a considerable scatter. Furthermore, the cone-point resistance, (the cone-point being small compared to a pile toe) may be misleadingly high in gravel and layered soils. Schmertmann [1978] has indicated an averaging procedure to be used for offsetting scatter, whether caused by natural (real) variation in the soil or inherent in the test.

The piezocone, which is a cone penetrometer equipped with pore pressure measurement devices at the point, is a considerable advancement on the static cone. By means of the piezocone, the cone information can be related more dependably to soil parameters and a more detailed analysis can be performed. Soil is variable, however, and the increased and more representative information obtained also means that a certain digestive judgment can and must be exercised to filter the data for computation of pile capacity. In other words, the designer is back to square one: more thoroughly informed and less liable to jump to false conclusions, but certainly not independent of site-specific experience. Eslami and Fellenius (1997) and Fellenius and Eslami (2000) have presented comprehensive information on soil profiling and analysis on pile capacity based on CPT data.

The Lambda Method

Vijayvergiya and Focht [1972] compiled a large number of results from static loading tests on essentially shaft-bearing piles in reasonably uniform soil and found that, for these test results, the mean unit shaft resistance is a function of depth and can be correlated to the sum of the mean overburden effective stress plus twice the mean undrained shear strength within the embedment depth, as follows.

$$r_s = \lambda(\sigma'_m + 2c_m) \quad (23.31)$$

where r_m = mean shaft resistance along the pile
λ = the lambda correlation coefficient
σ_m = mean overburden effective stress
c_m = mean undrained shear strength

The correlation factor is called "lambda" and it is a function of pile embedment depth, reducing with increasing depth, as shown in Table 23.5.

The lambda method is almost exclusively applied to determining the shaft resistance for heavily loaded pipe piles for offshore structures in relatively uniform soils.

TABLE 23.5 Approximate Values of λ

Embedment		λ
(ft)	(m)	(–)
0	0	0.50
10	3	0.36
25	7	0.27
50	15	0.22
75	23	0.17
100	30	0.15
200	60	0.12

Field Testing for Determining Axial Pile Capacity

The capacity of a pile is of most reliable value when determined in a full-scale field test. However, despite the numerous static loading tests that have been carried out and the many papers that have reported on such tests and their analyses, the understanding of static pile testing in current engineering practice leaves much to be desired. The reason is that engineers have concerned themselves with mainly one question — "Does the pile have a certain least capacity?" — finding little of practical value in analyzing the pile–soil interaction, the load transfer.

A static loading test is performed by loading a pile with a gradually or stepwise increasing force while monitoring the movement of the pile head. The force is obtained by means of a hydraulic jack reacting against a loaded platform or anchors.

The American Society for Testing and Materials, ASTM, publishes three standards, D-1143, D-3689, and D-3966, for static testing of a single pile in axial compression, axial uplift, and lateral loading, respectively. The ASTM standards detail how to arrange and perform the pile test. Wisely, they do not include how to interpret the tests, because this is the responsibility of the engineer in charge, who is the only one with all the site-and project-specific information necessary for the interpretation.

The most common test procedure is the slow maintained load method referred to as the "standard loading procedure" in the ASTM Designation D-1143 and D-3689, in which the pile is loaded in eight equal increments up to a maximum load, usually twice a predetermined allowable load. Each load level is maintained until zero movement is reached, defined as 0.25 mm/hr (0.01 in./hr). The final load, the 200 percent load, is maintained for a duration of 24 hours. The "standard method" is very time-consuming, requiring from 30 to 70 hours to complete. It should be realized that the words "zero movement" are very misleading: The "zero" movement rate mentioned is equal to a movement of more than 2 m (7 ft) per year!

Each of the eight load increments is placed onto the pile very rapidly; as fast as the pump can raise the load, which usually takes about 20 seconds to 2 minutes. The size of the load increment in the "standard procedure" — 12.5 percent of the maximum load — means that each such increase of load is a shock to the pile and the soil. Smaller increments that are placed more frequently disturb the pile less, and the average increase of load on the pile during the test is about the same. Such loading methods provide more consistent, reliable, and representative data for analysis.

Tests that consist of load increments applied at constant time intervals of 5, 10, or 15 minutes are called quick maintained-load tests or just "quick tests." In a quick test, the maximum load is not normally kept on the pile longer than any other load before the pile is unloaded. Unloading is done in about ten steps of no longer duration than a few minutes per load level. The quick test allows for applying one or more load increments beyond the minimum number that the particular test is designed for, that is, making use of the margin built into the test. In short, the quick test is, from the technical, practical, and economic points of view, superior to the "standard loading procedure."

A quick test should aim for 25 to 40 increments with the maximum load determined by the amount of reaction load available or the capacity of the pile. For routine cases, it may be preferable to stay at a maximum load of 200 percent of the intended allowable load. For ordinary test arrangements, where only the load and the pile head movement are monitored, time intervals of 10 minutes are suitable and allow for the taking of 2 to 4 readings for each increment. When testing instrumented piles, where the instruments take a while to read (scan), the time interval may have to be increased. To go beyond 20 minutes, however, should not be necessary. Nor is it advisable, because of the potential risk for influence of time-dependent movements, which may impair the test results. Usually, a quick test is completed within three to six hours.

In routine tests, cyclic loading or even single unloading and loading phases must be avoided, as they do little more than destroy the possibility of a meaningful analysis of the test results. There is absolutely no logic in believing that anything of value on load distribution and toe resistance can be obtained from an occasional unloading or from one or a few "resting periods" at certain load levels, when considering that we are testing a unit that is subjected to the influence of several soil types, is subjected to residual stress of unknown magnitude, exhibits progressive failure, etc., and when all we know is what is applied and measured at the pile head.

Interpretation of Failure Load

For a pile that is stronger than the soil, the failure load is reached when rapid movement occurs under sustained or slightly increased load (the pile plunges). However, this definition is inadequate, because plunging requires large movements. To be useful, a definition of failure load must be based on some mathematical rule and generate a repeatable value that is independent of scale relations and the opinions of the individual interpreter. Furthermore, it has to consider the shape of the load-movement curve or, if not, it must consider the length of the pile (which the shape of the curve indirectly does).

Fellenius [1975, 1980] compiled several methods used for interpreting failure or limit loads from a load-movement curve of a static loading test. The most well-known method is the offset limit method proposed by Davisson [1972]. This limit load is defined as the load corresponding to the movement that exceeds the elastic compression of the pile by an offset of 4 mm (0.15 inch) plus a value equal to the diameter of the pile divided by 120. It must be realized, however, that the offset limit load is a deformation limit that is determined taking into account the stiffness and length of the pile. It is not necessarily equal to the failure load of the pile.

The offset limit has the merit of allowing the engineer, when proof testing a pile for a certain allowable load, to determine *in advance* the maximum allowable movement for this load with consideration of the length and size of the pile. Thus, contract specifications can be drawn up including an acceptance criterion for piles proof tested according to quick-testing methods. The specifications can simply call for a test to at least twice the design load, as usual, and declare that at a test load equal to a factor F times the design load the movement shall be smaller than the Davisson offset from the elastic column compression of the pile. Normally, F would be chosen within a range of 1.8 to 2.0. The acceptance criterion could be supplemented with the requirement that the safety factor should also be smaller than a certain minimum value calculated on pile bearing failure defined according to the 80% criterion or other preferred criterion.

Influence of Errors

A static loading test is usually considered a reliable method for determining the capacity of a pile. However, even when using new manometers and jacks and calibrating them together, the applied loads is usually substantially overestimated. The error is usually about 10% to 15% of the applied load. Errors as large as 30% to 40% are not uncommon.

The reason for the error is that the jacking system is required to both provide the load and to measure it, and because load cells with moving parts are considerably less accurate than those without moving parts. For example, when calibrating testing equipment in the laboratory, one ensures that no eccentric loading, bending moments, or temperature variations influence the calibration. In contrast, all of these adverse factors are at hand in the field and influence the test results to an unknown extent, unless a load cell is used.

The above deals with the error of the applied load. The error in movement measurement can also be critical. Such errors do not originate in the precision of the reading — the usual precision is more than adequate — but in undesirable influences, such as heave or settlement of the reference beam during unloading the ground when loading the pile. For instance, one of the greatest spoilers of a loading test is the sun: The reference beam must be shielded from sunshine at all times.

Dynamic Analysis and Testing

The penetration resistance of driven piles provides a direct means of determining bearing capacity of a pile. In impacting a pile, a short-duration force wave is induced in the pile, giving the pile a downward velocity and resulting in a small penetration of the pile. Obviously, the larger the number of blows necessary to achieve a certain penetration, the stronger the soil. Using this basic principle, a large number of so-called pile-driving formulas have been developed for determining pile-bearing capacity. All these formulas are based on equalizing potential energy available for driving in terms of weight of hammer times its height of fall (stroke) with the capacity times penetration ("set") for the blow. The penetration value often includes a loss term.

The principle of the dynamic formulas is fundamentally wrong as wave action is neglected along with a number of other aspects influencing the penetration resistance of the pile. Nevertheless, pile-driving formulas have been used for many years and with some degree of success. However, success has been due less to the theoretical correctness of the particular formulas used and more to the fact that the users possessed adequate practical experience to go by. When applied to single-acting hammers, use of a dynamic formula may have some justification. However, dynamic formulas are the epitome of an outmoded level of technology and they have been or must be replaced by modern methods, such as the wave equation analysis and dynamic measurements, which are described below.

Pile-driving formulas or any other formula applied to vibratory hammers are based on a misconception. Vibratory driving works by eliminating resistance to penetration, not by overcoming it. Therefore, records of penetration combined with frequency, energy, amplitudes, and so on can relate only to the resistance not eliminated, not to the static pile capacity after the end of driving.

Pile-driving hammers are rated by the maximum potential energy determined as the ram weight times the maximum ram travel. However, diesel hammers and double-acting air/steam hammers, but also single-acting air/steam hammers, develop their maximum potential energy only during favorable combinations with the pile and the soil. Then, again, the energy actually transferred to the pile may vary due to variation in cushion properties, pile length, toe conditions, etc. Therefore, a relation between the hammer rated energy and measured transferred energy provides only very little information on the hammer.

For reliable analysis, all aspects influencing the pile driving and penetration resistance must be considered: hammer mass and travel, combustion in a diesel hammer, helmet mass, cushion stiffness, hammer efficiency, soil strength, viscous behavior of the soil, and elastic properties of the pile, to mention some. This analysis is made by means of commercially available wave equation programs, such as the GRLWEAP [GRL, 1993].

However, the parameters used as input into a wave equation program are really variables with certain ranges of values and the number of parameters included in the analysis is large. Therefore, the result of an analysis is only qualitatively correct, and not necessarily quantitatively correct, unless it is correlated to observations. The full power of the wave equation analysis is only realized when combined with dynamic measurements during pile driving by means of transducers attached to the pile head. The impact by the pile-driving hammer produces strain and acceleration in the pile which are picked up by the transducers and transmitted via a cable to a data acquisition unit (the Pile Driving Analyzer), which is placed in a nearby monitoring station. The complete generic field-testing procedure is described in the American Society for Testing and Materials, Standard for Dynamic Measurements, ASTM D-4945.

Dynamic testing, also called dynamic monitoring, is performed with the Pile Driving Analyzer (PDA). The PDA measurements provide much more information than just the value of the capacity of the pile, such as the energy transferred into the pile, the stresses in the pile, and the hammer performance. The dynamic data can be subjected to special analyses and provide invaluable information for determining that the piles are installed correctly, that the soil response is what was assumed in the design, and much more. For details, see Rausche et al. [1985] and Hannigan [1990]. Should difficulties develop with the pile driving at the site, the dynamic measurements can normally determine the reason for the difficulties and how to eliminate them. In the process, the frequent occurrence of having difficulties grow into a dispute between the contractor, the engineer, and the owner is avoided.

The dynamic measurements provide quantitative information of how the pile hammer functions, the compression and tension stresses that are imposed on the pile during the driving, and how the soil responds to the driving of the pile, including information pile static capacity. The dynamic measurements can also be used to investigate damage and defects in the pile, such as voids, cracks, spalling, local buckling, etc.

Dynamic records are routinely subjected to a detailed analysis called the CAPWAP signal matching analysis [Rausche et al., 1972]. The CAPWAP analysis provides, first of all, a calculated static capacity and the distribution of resistance along the pile. However, it also provides several additional data, for example, the movement necessary to mobilize the full shear resistance in the soil (the quake) and damping values for input in a wave equation analysis.

Pile Group Example: Axial Design

The design approach is illustrated in the following example: A group of 25 piles consisting of 355-mm diameter, closed-end steel pipes are to be driven at equal spacing and in a square configuration at a site with the soil profile equal to that described earlier as background to Table 23.1. The 1.5-m earth fill over 36 m^2 area will be placed symmetrically around the pile group. The pile cap is 9.0 m^2 and placed level with the ground surface.

Each pile will be subjected to dead and live loads of 800 kN and 200 kN, respectively. The soils investigation has established a range of values of the soil parameters necessary for the calculations, such as density, compressibility, consolidation coefficient, as well as the parameters (β and N_t) used in the effective stress calculations of load transfer. A load-transfer analysis is best performed using a range (boundary values) of β and N_t parameters, which differentiate upper and lower limits of reasonable values. The analysis must include several steps in approximately the following order.

Determine first the range of installation length (using the range of effective stress parameters) as based on the required at-least capacity, which is stated, say, to be at least equal to the sum of the loads times a factor of safety of 3.0: 3.0(800 + 200) = 3000 kN.

To obtain a capacity of 3000 kN, applying the lower boundary of β and N_t, the piles have to be installed to a penetration into the sandy till layer of 5 m, that is, to a depth of 32 m. Table 23.6 presents the results of the load-transfer calculations for this embedment depth. The calculations have been made with the Unipile program [Goudreault and Fellenius, 1990] and the results are presented in the format of a hand calculation to ease verifying the computer calculations. The precision indicated by stress values given with two decimals is to assist in the verification of the calculations and does not suggest a corresponding level of accuracy. Moreover, the effect of the 9 m^2 "hole" in the fill for the pile cap was ignored in the calculation. Were its effect to be included in the calculations, the calculated capacity would reduce by 93 kN or the required embedment length increase by 0.35 m.

The calculated values have been plotted in Fig. 23.5 in the form of two curves: a resistance curve giving the load transfer as in a static loading test to failure (3000 kN); and a load curve for long-term conditions, starting at the dead load of 800 kN and increasing due to negative skin friction to a maximum at the neutral plane. The load and resistance distributions for the example pile follow Eqs. (23.27) and (23.28).

Pile Group Settlement

The piles have reached well into the sand layers, which will not compress much for the increase of effective stress. Therefore, settlement of the pile group will be minimal and will not govern the design.

Installation Phase

The calculations shown above pertain to the service condition of the pile and are not quite representative for the installation (construction) phase. First, when installing the piles (these piles will be driven), the earth fill is not yet placed, which means that the effective stress in the soil is smaller than during the service conditions. More important, during the pile driving, large excess pore pressures are induced in the soft clay layer and, probably, also in the silty sand, which further reduces the effective stress. An analysis imposing increased pore pressures in these layers suggests that the capacity is about 2200 kN at the end of the initial driving (EOID) using the depth and effective stress parameters indicated in Table 23.6 for the 32 m installation depth. The subsequent dissipation of the pore pressures will result in an about 600-kN soil increase of capacity due to set-up. (The stress increase due to the earth fill will provide the additional about 200 kN to reach the 3000-kN service capacity.)

TABLE 23.6 Calculation of Pile Capacity

	Live Load, Q_l = 200 kN			Shaft Resistance, R_s = 1817 kN		
Area A_s = 1.115 m²/m	Dead Load, Q_d = 800 kN			Toe Resistance, R_t = 1205 kN		
Area A_t = 0.009 m²	Total Load = 1000 kN			Total Resistance, R_u = 3021 kN		
Factor of Safety = 3.2	Depth to Neutral Plane = 26.51 m			Load at Neutral Plane = 1911 kN		

Depth (m)	Total Stress (kPa)	Pore Pres. (kPa)	Eff. Stress (kPa)	Incr. R_s (kN)	$Q_d + Q_n$ (kN)	$Q_u - R_s$ (kN)
		Layer 1 Sandy Silt ρ = 2000 kg/m³ β = 0.40				
0.00	30.00	0.00	30.00	0.0	800	3232
1.00 (GWT)	48.40	0.00	48.40	17.5	817	3215
4.00	104.30	30.00	74.30	82.1	900	3133
		Layer 2 Soft Clay ρ = 1700 kg/m³ β = 0.30				
4.00	104.30	30.00	74.30		900	3133
5.00	120.13	43.53	76.60	25.2	925	3108
6.00	136.04	57.06	78.98	26.0	951	3082
7.00	152.03	70.59	81.44	26.8	978	3055
8.00	168.08	84.12	83.96	27.7	1005	3027
9.00	184.20	97.65	86.55	28.5	1034	2999
10.00	200.37	111.18	89.20	29.4	1063	2969
11.00	216.60	124.71	91.89	30.3	1094	2939
12.00	232.88	138.24	94.64	31.2	1125	2908
13.00	249.19	151.76	97.43	32.1	1157	2876
14.00	265.55	165.29	100.26	33.1	1190	2842
15.00	281.95	178.82	103.12	34.0	1224	2808
16.00	298.38	192.35	106.03	35.0	1259	2773
17.00	314.84	205.88	108.96	36.0	1295	2737
18.00	331.33	219.41	111.92	37.0	1332	2701
19.00	347.85	232.94	114.91	37.9	1370	2663
20.00	364.40	246.47	117.93	39.0	1409	2624
21.00	380.97	260.00	120.97	40.0	1449	2584
		Layer 3 Silty Sand ρ = 2100 kg/m³ β = 0.50				
21.00	380.97	260.00	120.97		1449	2584
22.00	401.56	270.00	131.56	70.4	1519	2513
23.00	422.17	280.00	142.17	76.3	1596	2437
24.00	442.80	290.00	152.80	82.2	1678	2355
25.00	463.45	300.00	163.45	88.2	1766	2267
26.00	484.11	310.00	174.11	94.1	1860	2172
27.00	504.80	320.00	184.80	100.1	1960	2072
		Layer 4 Ablation Till ρ = 2100 kg/m³ β = 0.50				
27.00	504.80	320.00	184.80		1960	2072
30.00	569.93	350.00	219.93	372.4	2332	1700
32.00	613.41	370.00	243.41	285.1	2617	1205
						N_t = 50

Note: Calculations by means of UNIPILE.

The design must include the selection of the pile-driving hammer, which requires the use of software for wave equation analysis, called WEAP analysis [Goble et al., 1980; GRL, 1993; Hannigan, 1990]. This analysis requires input of soil resistance in the form as result of static load-transfer analysis. For the installation (initial driving) conditions, the input is calculated considering the induced pore pressures. For restriking conditions, the analysis should consider the effect of soil set-up.

By means of wave equation analysis, pile capacity at initial driving — in particular the EOID — and restriking (RSTR) can be estimated. However, the analysis also provides information on what driving

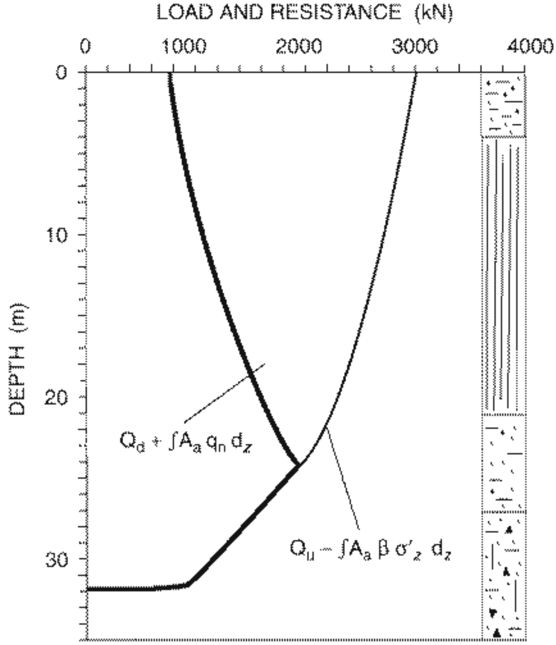

FIGURE 23.5 Load-transfer and resistance curves.

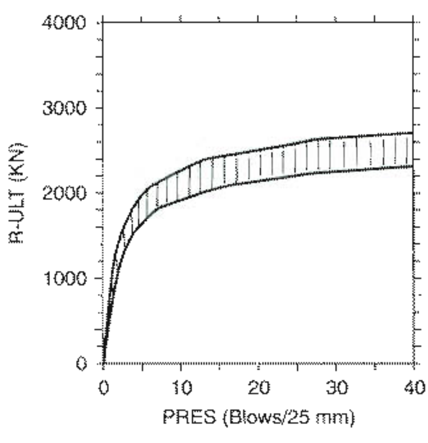

FIGURE 23.6 Bearing graph from WEAP analysis.

stresses to expect, indeed, even the length of time and the number of blows necessary to drive the pile. The most commonly used result is the bearing graph, that is, a curve showing the ultimate resistance (capacity) versus the penetration resistance (blow count) as illustrated in Fig. 23.6. As in the case of the static analysis, the parameters to input to a wave equation analysis can vary within upper and lower limits, which results in not one curve but a band of curves within envelopes as shown in Fig. 23.6. The input parameters consist of the particular hammer to use with its expected efficiency, the static resistance variation, dynamic parameters for the soil, such as damping and quake values, and many other parameters. It should be obvious that no one should expect a single answer to the analysis. Figure 23.6 shows that at EOID for the subject example, when the predicted capacity is about 2200 kN, the penetration resistance (PRES) will be about 10 blows/25 mm through about 20 blows/25 mm.

Notice that the wave equation analysis postulates observation of the actual penetration resistance when driving the piles, as well as a preceding static analysis. Then, common practice is to combine the analyses with a factor of safety ranging from 2.5 through 3.0.

Figure 23.6 demonstrates that the hammer selected for the driving cannot drive the pile against the 3000-kN capacity expected after full set-up. That is, restriking cannot prove out the capacity. This is a common occurrence. Bringing in a larger hammer may be a costly proposition. It may also be quite unnecessary. If the soil profile is well known, the static analysis correlated to the soil profile and to careful observation during the entire installation driving for a few piles, sufficient information is usually obtained to support a satisfactory analysis of the pile capacity and load transfer. That is, the capacity after set-up is inferred and sufficient for the required factor of safety.

When conditions are less consistent, when savings may result, and when safety otherwise suggests it to be good practice, the pile capacity is tested directly. Conventionally, this is accomplished by means of a static loading test. Since about 1975, dynamic tests have likewise often been performed. Static tests are costly and time-consuming, and are therefore usually limited to one or a few piles. In contrast, dynamic tests can be obtained quickly and economically and can be performed on several piles, thus providing assurance in numbers. For larger projects, static and dynamic tests are often combined. More recently, a new testing method called Statnamic has been proposed [Bermingham and Janes, 1989]. The Statnamic method is particularly intended for high-capacity bored piles (drilled piers). The Osterberg O-Cell (Fellenius, 2001) is a very useful new tool for the geotechnical engineer to use when reliable separation of shaft and toe resistances is required. The O-cell is suitable for testing of both bored and drive piles.

When the capacity is determined by direct testing, the factor of safety of the design is reduced. The usual range is from 2.0 to 2.2. When the design and the installation are tested by means of static or dynamic proof testing on the site, the factor of safety is often reduced to the range of 1.8 through 2.0. Notice that results of the tests must not just be given in terms of a capacity value (which can vary depending on how the ultimate resistance is defined); the load transfer should also be included in the analysis.

All analyses for a project must apply the same factor of safety. Therefore, for the subject example, when the designer knows that the capacity will be verified by means of a direct test, the minimum factor of safety to apply reduces to about 2.2, say. That is, the piles need only be driven to a final (after set-up) capacity of 2000 kN or 2200 kN. Considering the initial driving conditions, the capacity at EOID could be limited to about 1600 kN and this should be obtainable at a penetration into the till of slightly less than one meter and a penetration resistance of 4 to 5 blow/25 mm. Then, the subsequent increase due to set-up to 2200 kN is verified in a dynamic of static test *combined* with restriking to verify that the PRES values have increased to beyond about 10 blows/25 mm.

Summary of Axial Design of Piles

In summary, pile design consists of the following steps.

1. Compile soil data and perform a static analysis of the load transfer.
2. Verify that the ultimate pile resistance (capacity) is at least equal to the factor of safety times the sum of the dead and the live load (do not include the dragload in this calculation).
3. Verify that the maximum load in the pile, which is the sum of the dead load and the dragload is smaller than the structural strength of the pile divided by the appropriate factor of safety (usually 1.5) times. (Do not include the live load in this calculation.)
4. Verify that the pile group settlement does not exceed the maximum deformation permitted by the structural design.
5. Perform wave equation analysis to select the pile-driving hammer and to decide on the driving and termination criteria (for driven piles).
6. Observe carefully the pile driving (construction) and verify that the work proceeds as anticipated. Document the observations (that is, keep a complete and carefully prepared log!).
7. When the factor of safety needs to be 2.5 or smaller, verify pile capacity by means of static or dynamic testing.

Design of Piles for Horizontal Loading

Because foundation loads act in many different directions depending on the load combination, piles are rarely loaded in true axial direction only. Therefore, a more or less significant lateral component of the total pile load always acts in combination with an axial load. The imposed lateral component is resisted by the bending stiffness of the pile and the shear resistance mobilized in the soil surrounding the pile.

An imposed horizontal load can also be carried by means of inclined piles, if the horizontal component of the axial pile load is at least equal to and acting in the opposite direction to the imposed horizontal load. Obviously, this approach has its limits, as the inclination cannot be impractically large. It should, preferably, not be greater than 4 (vertical) to 1 (horizontal). Also, only one load combination can provide the optimal lateral resistance.

In general, it is not correct to resist lateral loads by means of combining the soil resistance for the piles (inclined as well as vertical) with the lateral component of the vertical load for the inclined piles. The reason is that resisting an imposed lateral load by means of soil shear requires the pile to move against the soil. The pile will rotate due to such movement and an inclined pile will then either push up against or pull down from the pile cap, which will substantially change the axial load in the pile.

In design of vertical piles installed in a homogeneous soil and subjected to horizontal loads, an approximate and usually conservative approach is to assume that each pile can sustain a horizontal load equal to the passive earth pressure acting on an equivalent wall with depth of $6b$ and width $3b$, where b is the pile diameter or face-to-face distance [Canadian Geotechnical Society, 1985].

Similarly, the lateral resistance of a pile group may be approximated by the soil resistance on the group calculated as the passive earth pressure over an equivalent wall with depth equal to $6b$ and width equal to:

$$L_e = L + 2b \tag{23.32}$$

where L_e = equivalent width
L = the length, center-to-center, of the pile group in plan perpendicular to the direction of the imposed loads
b = the width of the equivalent area of the group in plan parallel to the direction of the imposed loads

The lateral resistance calculated according to Eq. (23.32) must not exceed the sum of the lateral resistance of the individual piles in the group. That is, for a group of n piles, the equivalent width of the group, L_e, must be smaller than n times the equivalent width of the individual pile, $6b$. For an imposed load not parallel to a side of the group, calculate for two cases, applying the components of the imposed load that are parallel to the sides.

The very simplified approach expressed above does not give any indication of movement. Nor does it differentiate between piles with fixed heads and those with heads free to rotate; that is, no consideration is given to the influence of pile bending stiffness. Because the governing design aspect with regard to lateral behavior of piles is lateral displacement, and the lateral capacity or ultimate resistance is of secondary importance, the usefulness of the simplified approach is very limited in engineering practice.

The analysis of lateral behavior of piles must involve two aspects:

1. *Pile response.* The bending stiffness of the pile, how the head is connected (free head, or fully or partially fixed head).
2. *Soil response.* The input in the analysis must include the soil resistance as a function of the magnitude of lateral movement.

The first aspect is modeled by treating the pile as a beam on an "elastic" foundation, which is accomplished by solving a fourth-degree differential equation with input of axial load on the pile, material properties of the pile, and the soil resistance as a nonlinear function of the pile displacement.

The derivation of lateral stress may make use of a simple concept called *coefficient of subgrade reaction* having the dimension of force per volume [Terzaghi, 1955]. The coefficient is a function of the soil density or strength, the depth below the ground surface, and the diameter (side) of the pile. In cohesionless soils, the following relation is used:

$$k_s = n_h \frac{z}{b} \tag{23.33}$$

where k_s = coefficient of horizontal subgrade reaction
n_h = coefficient related to soil density
z = depth
b = pile diameter

The intensity of the lateral stress, p_z, mobilized on the pile at depth z is then as follows:

$$p_z = k_s y_z b \tag{23.34}$$

where y_z = the horizontal displacement of the pile at depth z. Combining Eqs. (23.33) and (23.34), we get

$$p_z = n_h y_z z. \tag{23.35}$$

The relation governing the behavior of a laterally loaded pile is then as follows:

$$Q_h = EI \frac{d^4y}{dx^4} + Q_v \frac{d^2y}{dx^2} - p \tag{23.36}$$

where Q_h = lateral load on the pile
EI = bending stiffness (flexural rigidity)
Q_v = axial load on the pile

Design charts have been developed that, for an input of imposed load, basic pile data, and soil coefficients, provide values of displacement and bending moment. See, for instance, the *Canadian Foundation Engineering Manual* [Canadian Geotechnical Society, 1985].

The design charts cannot consider all the many variations possible in an actual case. For instance, the $p-y$ curve can be a smooth rising curve, can have an ideal elastic-plastic shape, or can be decaying after a peak value. As an analysis without simplifying shortcuts is very tedious and time-consuming, resort to charts has been necessary in the past. However, with the advent of the personal computer, special software has been developed, which makes the calculations easy and fast. In fact, as in the case of pile-driving analysis and wave equation programs, engineering design today has no need for computational simplifications. Exact solutions can be obtained as easily as approximate ones. Several proprietary and public domain programs are available for analysis of laterally loaded piles.

One must not be led to believe that, because an analysis is theoretically correct, the results also predict the true behavior of the pile or pile group. The results must be correlated to pertinent experience and, lacking this, to a full-scale test at the site. If the experience is limited and funds are lacking for a full-scale correlation test, then a prudent choice is necessary of input data, as well as of margins and factors of safety.

Designing and analyzing a lateral test is much more complex than for the case of axial behavior of piles. In service, a laterally loaded pile almost always has a fixed-head condition. However, a fixed-head test is more difficult and costly to perform as opposed to a free-head test. A lateral test without inclusion of measurement of lateral deflection down the pile (bending) is of limited value. While an axial test should not include unloading cycles, a lateral test should be a cyclic test and include a large number of cycles at different load levels. The laterally tested pile is much more sensitive to the influence of neighboring piles than is the axially tested pile. Finally, the analysis of the test results is very complex and requires the use of a computer and appropriate software.

Seismic Design of Lateral Pile Behavior

Seismic design of lateral pile behavior is often taken as being the same as the conventional lateral design. A common approach is to assume that the induced lateral force to be resisted by piles is static and equal to a proportion, usually 10% of the vertical force acting on the foundation. If all the horizontal force is designed to be resisted by inclined piles, and all piles — including the vertical ones — are designed to resist significant bending at the pile cap, this approach is normally safe, albeit costly.

The seismic wave appears to the pile foundation as a soil movement forcing the piles to move with the soil. The movement is resisted by the pile cap; bending and shear are induced in the piles; and a horizontal force develops in the foundation, starting it to move in the direction of the wave. A half period later, the soil swings back, but the foundation is still moving in the first direction, and therefore the forces increase. This situation is not the same as the one originated by a static force.

Seismic lateral pile design consists of determining the probable amplitude and frequency of the seismic wave as well as the natural frequency of the foundation and structure supported by the piles. The first requirement is, as in all seismic design, that the natural frequency must not be the same as that of the seismic wave. Then the probable maximum displacement, bending, and shear induced at the pile cap are estimated. Finally the pile connection and the pile cap are designed to resist the induced forces.

There is at present a rapid development of computer software for use in detailed seismic design.

Defining Terms

Capacity — The maximum or ultimate soil resistance mobilized by a foundation unit.

Capacity, bearing — The maximum or ultimate soil resistance mobilized by a foundation unit subjected to downward loading.

Dragload — The load transferred to a deep foundation unit from negative skin friction.

Factor of safety — The ratio of maximum available resistance or of the capacity to the allowable stress or load.

Foundation — A system or arrangement of structural members through which the loads are transferred to supporting soil or rock.

Groundwater table — The upper surface of the zone of saturation in the ground.

Load, allowable — The maximum load that may be safely applied to a foundation unit under expected loading and soil conditions and determined as the capacity divided by the factor of safety.

Neutral plane — The location where equilibrium exists between the sum of downward acting permanent load applied to the pile and dragload due to negative skin friction and the sum of upward acting positive shaft resistance and mobilized toe resistance. The neutral plane is also where the relative movement between the pile and the soil is zero.

Pile — A slender deep foundation unit, made of wood, steel, concrete, or combinations thereof, which is either premanufactured and placed by driving, jacking, jetting, or screwing, or cast *in situ* in a hole formed by driving, excavating, or boring. A pile can be a non-displacement, low-displacement, or displacement type.

Pile head — The uppermost end of a pile.

Pile point — A special type of pile shoe.

Pile shaft — The portion of the pile between the pile head and the pile toe.

Pile shoe — A separate reinforcement attached to the pile toe of a pile to facilitate driving, to protect the lower end of the pile, and/or to improve the toe resistance of the pile.

Pile toe — The lowermost end of a pile. (Use of terms such as pile tip, pile point, or pile end in the same sense as pile toe is discouraged).

Pore pressure — Pressure in the water and gas present in the voids between the soil grains minus the atmospheric pressure.

Pore pressure, artesian — Pore pressure in a confined body of water having a level of hydrostatic pressure higher than the level of the ground surface.

Pore pressure, hydrostatic — Pore pressure varying directly with a free-standing column of water.

Pore pressure elevation, phreatic — The elevation of a groundwater table corresponding to a hydrostatic pore pressure equal to the actual pore pressure.

Pressure — Omnidirectional force per unit area. (Compare stress.)

Settlement — The downward movement of a foundation unit or soil layer due to rapidly or slowly occurring compression of the soils located below the foundation unit or soil layer, when the compression is caused by an increase of effective stress.

Shaft resistance, negative — Soil resistance acting downward along the pile shaft because of an applied uplift load.

Shaft resistance, positive — Soil resistance acting upward along the pile shaft because of an applied compressive load.

Skin friction, negative — Soil resistance acting downward along the pile shaft as a result of downdrag and inducing compression in the pile.

Skin friction, positive — Soil resistance acting upward along the pile shaft caused by swelling of the soil and inducing tension in the pile.

Stress — Unidirectional force per unit area. (Compare pressure.)

Stress, effective — The total stress in a particular direction minus the pore pressure.

Toe resistance — Soil resistance acting against the pile toe.

References

Altaee, A., Evgin, E., and Fellenius, B. H. 1992. Axial load transfer for piles in sand. I: Tests on an instrumented precast pile. *Can. Geotech. J.* 29 (1):11–20.

Altaee, A., Evgin, E., and Fellenius, B. H. 1993. Load transfer for piles in sand and the critical depth. *Can. Geotech. J.* 30(2):465–463.

American Society of Civil Engineers. 1989. *Proc. Cong. Found. Eng., Geotech. Eng. Div., ASCE.* Evanston, IL.

American Society of Civil Engineers. 1991. *Proc. Geotech. Eng. Cong. ASCE.* Boulder, CO.

American Society of Civil Engineers. 1994. Geotechnical Engineering Division. *Prof. Spec. Conf. Vert. Hor. Deform. Found. Embank, ASCE.* Houston, TX.

Bermingham, P., and Janes, M. 1989. An innovative approach to loading tests on high capacity piles. *Proc. Int. Conf. Piling Deep Found.,* 1:409–427. A. A. Balkema, Rotterdam.

Canadian Geotechnical Society, 1985. *Canadian Foundation Engineering Manual* (2nd ed.). BiTech, Vancouver.

Davisson, M. T., 1972. High capacity piles. *Proc. Innov. in Found. Const., ASCE.* Illinois Section. Chicago, pp. 81–112.

Eslami, A. and Fellenius, B. H. 1997. Pile capacity by direct CPT and CPTu methods applied to 102 case histories. *Can. Geotech. J.,* 34(6), 886–904.

Fang, H.-Y. 1991. *Foundation Engineering* (2nd ed.). Van Nostrand Reinhold, New York.

Fellenius, B. H. 1975. Test loading of piles. Methods, interpretation and new proof testing procedure. *J. Geotech. Eng., ASCE.* 101(GT9):855–869.

Fellenius, B. H. 1980. The analysis of results from routine pile loading tests. *Ground Engineering,* 13(6):19–31.

Fellenius, B. H. 1984. Ignorance is bliss — And that is why we sleep so well. *Geot. News, Can. Geotech. Soc. and the U.S. Nat. Soc. of the Int. Soc. of Soil Mech. and Found. Eng.* 2(4):14–15.

Fellenius, B. H. 1989. Tangent modulus of piles determined from strain data. *Proc. 1989 Found. Cong., Geotech. Div., ASCE.* 1:500–510.

Fellenius, B. H. 2001. The O-Cell — An innovative engineering tool. *Geotech. News Mag.,* 19(6), 55–58.

Fellenius, B. H. and Eslami, A. 2000. Soil profile interpreted from CPTu data. Proceedings of Year 2000 Geotechnics Conference, Southeast Asia Geotechnical Society, Asian Institute of Technology, Bangkok, Thailand, November 27–30, 2000, Balasubramaniam, A. S., Bergado, D. T., Der-Gyey, L., Seah, T. H., Miura, K., Phien-wej, N., and Nutalaya, P., Eds., Vol. 1, pp. 163–171.

Goble, G. G., Rausche, F., and Likins, G. 1980. The analysis of pile driving — a state-of-the-art. *Proc. 1st Int. Sem. of the Appl. Stress-wave Theory to Piles, Stockholm.* A. A. Balkema, Rotterdam, pp. 131–161.

Goudreault, P. A., and Fellenius, B. H. 1990. *Unipile Version 1.02 Users Manual.* Unisoft Ltd., Ottawa, p. 76.

Goudreault, P. A., and Fellenius, B. H. 1993. *Unisettle Version 1.1 Users Manual.* Unisoft Ltd., Ottawa, p. 58.

GRL, 1993. *Background and Manual on GRLWEAP Wave Equation Analysis of Pile Driving.* Goble, Rausche, Likins, Cleveland, OH.

Hannigan, P. J. 1990. *Dynamic Monitoring and Analysis of Pile Foundation Installations.* Deep Foundation Institute, Sparta, NJ, p. 69.

Holtz, R. D., and Kovacs, W. D. 1981. *An Introduction to Geotechnical Engineering.* Prentice Hall, New York, p. 780.

Ismael, N. F. 1985. Allowable bearing pressure from loading tests on Kuwaiti soils. *Can. Geotech. J.* 22(2):151–157.

Janbu, N. 1963. Soil compressibility as determined by oedometer and triaxial tests. *Proc. Eur. Conf. Soil Mech. Found. Eng.*, Wiesbaden. 1:19–25, 2:17–21.

Janbu, N. 1965. Consolidation of clay layers based on non-linear stress-strain. *Proc. 6th Int. Conf. on Soil Mech, Found. Eng.*, Montreal, 2:83–87.

Janbu, N. 1967. Settlement calculations based on the tangent modulus concept. *University of Trondheim, Nor. Inst. of Tech., Geotech. Inst. Bull.* 2:57.

Kany, M. 1959. *Beitrag zur berechnung von flachengrundungen.* Wilhelm Ernst und Zohn, Berlin, p. 201.

Ladd, C. C. 1991. Stability evaluation during staged construction. The twenty-second Terzaghi lecture. *J. of Geotech. Eng., ASCE.* 117(4):540–615.

Lambe, T. W., and Whitman, R. V. 1979. *Soil Mechanics.* John Wiley & Sons, New York.

Meyerhof, G. G. 1976. Bearing capacity and settlement of pile foundations. The eleventh Terzaghi Lecture. *J. of Geotech. Eng., ASCE.* 102(GT3):195–198.

Mitchell, J. K. 1976. *Fundamentals of Soil Behavior.* John Wiley & Sons, New York.

Newmark, N. M. 1935. Simplified computation of vertical stress below foundations. *Univ. Illinois, Eng. Expnt. Stat. Circ.*, 24.

Newmark, N. M. 1942. Influence chart for computation of stresses in elastic foundations. *Univ. Illinois, Eng. Expnt. Stat. Bull. 338*, 61(92).

Nordlund, R. L. 1963. Bearing capacity of piles in cohesionless soils. *J. Geotech. Eng., ASCE.* 89(SM3):1–35.

Rausche, F., Moses, F., and Goble. G. G. 1972. Soil resistance predictions from pile dynamics. *J. Geotech. Eng., ASCE.* 98(SM9):917–937.

Rausche, F., Goble, G. G., and Likins, G. E. 1985. Dynamic determination of pile capacity. *J. of the Geotech. Eng. ASCE.* 111(3):367–383.

Schmertmann, J. H. 1978. *Guidelines for Cone Penetration Test, Performance, and Design.* U.S. Federal Highway Administration, Washington, Report FHWA-TS-78-209, p. 145.

Taylor, D. W. 1948. *Fundamentals of Soil Mechanics.* Wiley & Sons, New York.

Terzaghi, K. 1955. Evaluation of coefficients of subgrade reaction. *Geotechnique.* 5(4):297–326.

Vesic, A. S. 1967. *A Study of Bearing Capacity of Deep Foundations.* Final Report Project B-189, Geor. Inst. of Tech., Engineering Experiment Station, Atlanta, GA, p. 270.

Vijayvergiya, V. N., and Focht, J. A., Jr. 1972. A new way to predict the capacity of piles in clay. *Proc. 4th Ann. Offshore Tech. Conf.* 2:865–874.

Winterkorn, H. F., and Fang, H.-Y. 1975. *Foundation Engineering* (1st ed.). Van Nostrand Reinhold, New York.

Further Information

Foundation Engineering Handbook. (1st ed.). 1975. Edited by Winterkorn, H. F., and Fang, H.-Y. Van Nostrand Reinhold, New York.

Foundation Engineering Handbook. (2nd ed.). 1991. Edited by Fang, H.-Y., Van Nostrand Reinhold, New York.

American Society of Civil Engineers, Geotechnical Engineering Division, *Congress on Foundation Engineering.* F. H. Kulhawy, editor. Evanston, IL, June 1989, Vols. 1 and 2.

American Society of Civil Engineers, *Geotechnical Engineering Congress.* F. G. McLean, editor. Boulder, CO, June 1991, Vols. 1 and 2.

American Society of Civil Engineers, Geotechnical Engineering Division, *Speciality Conference on Vertical and Horizontal Deformations for Foundations and Embankments,* Houston, June 1994, Vols. 1 and 2.

Lambe, T. W., and Whitman, R. V. 1979. *Soil Mechanics.* Series in Soil Engineering. John Wiley & Sons, New York.

Mitchell, J. K. 1976. *Fundamentals of Soil Behavior.* Series in Soil Engineering. John Wiley & Sons, New York.

Taylor, D. W. 1948. *Fundamentals of Soil Mechanics.* John Wiley & Sons, New York.